Average incremental cost pricing in electricity auctions

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MAX-PLANCK-GESELLSCHAFT

forward market design

LiDAM

CORE | Louvain Institute of Data Analysis and Modeling in economics and statistics

Content

Motivations and policy context (1)

- Electricity wholesale markets are typically organized as a **sealed-bid auction** with **uniform pricing**
- Complex bidding format: typically **include non-convex bids**
  - Express cost and constraints of production
  - *Substitution*, but also *complementarities* in the production of electricity, which make the problem complicated (Milgrom, 2017)
  - Unit commitment model in the US (start-up costs, minimum output, etc.)
  - Block orders in EU market (all-or-nothing production, minimum acceptance ratio, exclusive groups)
- Main implication of these non-convexities: **equilibrium is not guaranteed to exist** (Debreu, 1959) although it *might* exist in some cases (Bikhchandani and Mamer, 1997)

  → Might be impossible to find a price-allocation pair which is an equilibrium

  → This paper aims at addressing this issue

Motivations and policy context (2)

- Problem encountered in ***all* electricity markets** in the US and in EU
  - No equilibrium → pricing rule not obvious
  - **Heterogeneous pricing policies** implemented by electricity auctioneers
  - These policies have been **evolving** for the past 20 years
- **US markets** pricing policies (EPRI, 2019)
  - **1992** : Energy Policy Act (kickoff of electricity market liberalization)
  - **Early 2000'**: marginal pricing, with discriminatory side-payments, adopted by many ISOs
  - **2014**: FERC launched consultation about price formation (FERC, 2014)
  - **2015** : MISO implemented "extended" LMP
  - **2017** : PJM made similar proposal (PJM, 2017)
- **EU markets** pricing policies (Meeus, 2020)
  - **1996, 2003, 2009** : First, Second and Third Energy Packages
  - **2006**: Trilateral Market Coupling (BE-FR-NL, block orders and no-PAB rule) (Belpex et al., 2006)
  - **2014**: Single Day-Ahead Coupling (SDAC)

Pricing solutions to this problem

- In economics, it relates to how to price a commodity in presence of **fix costs**
  - *Inflate* the commodity **price above marginal cost** (e.g. Ramsey-Boiteux pricing)
  - *Complement* the uniform price with side-payments — **multi-part pricing** (Coase, 1946)
- In practice, electricity auctioneers often rely on a combination of both approaches
  - Electricity price set **above marginal cost**
  - Pay discriminatory **"make-whole" payments**
- 3 main pricing options (active field of research for the past 20 years):
  - **Marginal pricing** (O'Neill et al., 2005): price at marginal cost and pay (maybe a lot) of discriminatory side-payments
  - **Average incremental cost (AIC) pricing** (Bichler et al., 2022 ; Madani and Papavasiliou, 2022 ; O'Neill et al., 2023; Chen et al., 2024): price at the average incremental cost to eliminate the need of make-whole payments.
  - **Convex Hull Pricing (CHP)** (Hogan and Ring, 2003 ; Gribik et al., 2007 ; Schiro et al., 2015 ; Hua and Baldick, 2017; Chao, 2019 ; Stevens et al., 2024): inflate price above marginal cost but not to the extent to eliminate make-whole payments

Average incremental cost pricing

Price equals the highest average cost of online unit → all suppliers make non-negative profit

- Convex supplier **S1** & Non-convex supplier **S2**
  - [0, 30] MW
  - MC=10€/MWh
  - {0 ; [90, 100]} MW
  - MC=20€/MWh & NLC=1000€
- Demand = 110MW (fully inelastic)
- Optimal allocation: 20MW (S1) + 90MW (S2)
- Marginal cost price = 10€/MWh
- Average cost price = 31.11€/MWh

→ <u>**Objective of the paper:**</u>

- ***Formalize*** AIC pricing
- ***Theoretical properties***: understand consequences for market participants
- ***Numerical analysis*** with realistic auction dataset: get a sense of the order of magnitude & compare it with alternative pricing mechanisms

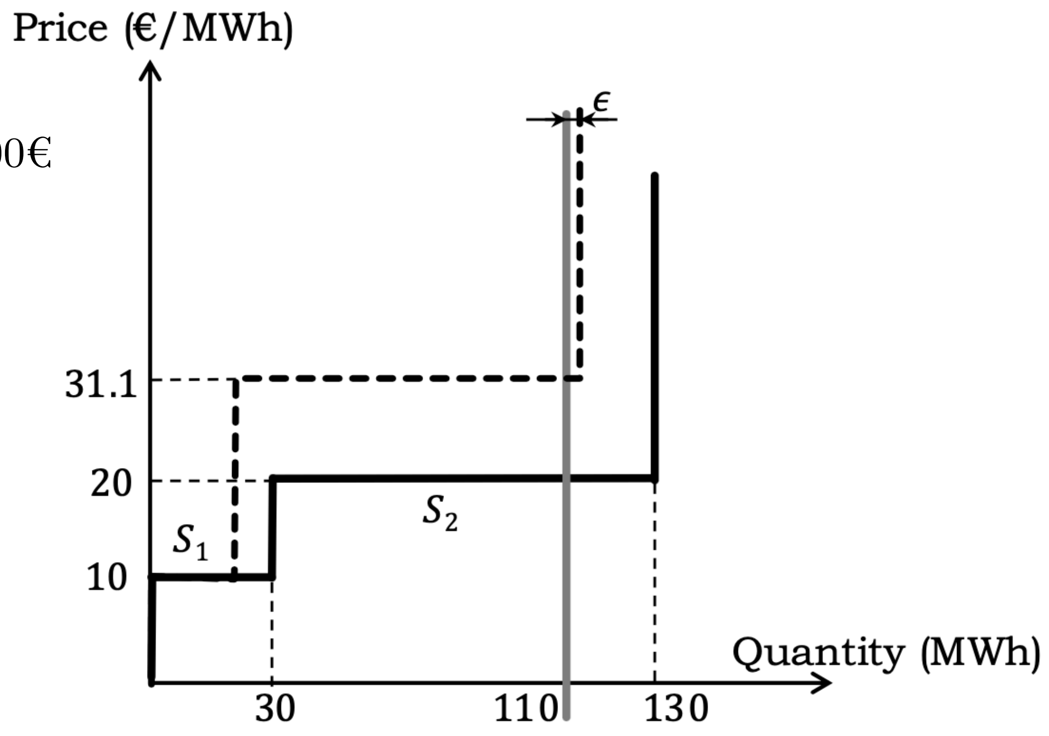

Figure 1: Aggregate marginal cost curve in Example 1.

Content

Electricity market model

$$z^* = \min_{c,q,x,f} \sum_{g \in \mathcal{G}} c_g \tag{1a}$$

Minimize the cost

$$\sum_{g \in \mathcal{G}_i} q_{g,t} - D_t^i = \sum_{\substack{l \in \\ from(i)}} f_{l,t} - \sum_{\substack{l \in \\ to(i)}} f_{l,t} \ \forall i \in \mathcal{N}, t \in \mathcal{T} \tag{1b}$$

Market-clearing constraints

$$(c, q, x)_g \in \mathcal{X}_g \ \ \forall g \in \mathcal{G} \tag{1c}$$

Production const. (producers' bids)

$$f \in \mathcal{F} \tag{1d}$$

Network constraints

<u>Assumptions</u>:

- Producers are price-taker
- Demand is fully inelastic

<u>2 main outputs</u>

- Allocation: (c^*, q^*, x^*, f^*)
- Price: π

How to measure distance to an equilibrium?
Lost opportunity costs (LOC)

Definition 2 (Lost Opportunity Cost) *Lost opportunity cost (LOC) is the difference between the maximum profit and the as-cleared profit under price π. It is defined hereafter for each supplier g (eq. (5)), for the network (eq. (6)) and in total (eq. (7)).*

$$LOC_g^{gen}(\pi) = \max_{\substack{(c,q,x)_g \\ \in \mathcal{X}_g}} \mathcal{P}_g(c, q, x, \pi) - \mathcal{P}_g(c^*, q^*, x^*, \pi) \quad (5)$$

$$LOC^{net}(\pi) = \max_{f \in \mathcal{F}} \mathcal{P}_N(f, \pi) - \mathcal{P}_N(f^*, \pi) \quad (6)$$

$$LOC(\pi) = \sum_{g \in \mathcal{G}} LOC_g^{gen}(\pi) + LOC^{net}(\pi) \quad (7)$$

How much each agent wants to deviate from market instructions

→ Lost opportunity costs relate to the **self-scheduling** problem

How to measure distance to an equilibrium?
Revenue shortfall (RS)

Definition 1 (Revenue Shortfall) *Revenue shortfall (RS) corresponds to the payments that are required in order to ensure a non-negative profit. It is defined for each supplier (eq. (2)), for the network (eq. (3)) and in total (eq. (4)).*

$$RS_g^{gen}(\pi) = -\min\left(0,\ \mathcal{P}_g(c^*, q^*, x^*, \pi)\right) \tag{2}$$

$$RS^{net}(\pi) = -\min\left(0,\ \mathcal{P}_N(f^*, \pi)\right) \tag{3}$$

$$RS(\pi) = \sum_{g \in \mathcal{G}} RS_g^{gen}(\pi) + RS^{net}(\pi) \tag{4}$$

How much additional side-payments each agent needs (on top of the uniform price) to break even

→ Revenue shortfalls relate to **make-whole payments**

→ Important relationship between LOC and RS: *RS is a specific type of LOC, where the "lost opportunity" is to exit the market* (to self-schedule at 0)

Several pricing candidates
The 3 cardinal points

<table>
<tr><th>Pricing scheme</th><th>Objective</th><th>Math. Formulation</th><th>Computation</th><th>References</th></tr>
<tr><td>Marginal Pricing</td><td></td><td>Fix binary variables (primal-dependent)</td><td>Easy</td><td>O’Neill et al. (2005)</td></tr>
<tr><td>Convex Hull Pricing</td><td>Minimize “Lost Opportunity Costs”</td><td>Take convex hull of production & consumption sets (primal-dual separated)</td><td>Difficult, but feasible (Stevens and Papavasiliou, 2022)</td><td>Hogan and Ring (2003) ; Gribik et al. (2007) ; Stevens et al. (2024)</td></tr>
<tr><td>Minimal Make-Whole Payment pricing</td><td rowspan="2">Minimize “Revenue Shortfall”</td><td>Solve ad-hoc problem (primal-dependent)</td><td>Easy</td><td>Bichler et al. (2022) ; Madani and Papavasiliou, 2022</td></tr>
<tr><td>Average incremental cost pricing</td><td>Take convex relaxation, then convex restriction of the problem (primal-dependent)</td><td>Easy</td><td>O’Neill et al., 2023 ; Chen et al., 2024</td></tr>
</table>

Content

1 *Introduction: pricing with non-convexities*

2 *The model*

3 *AIC pricing*

4 *Conclusions and discussion*

Average incremental cost pricing
Formal definition

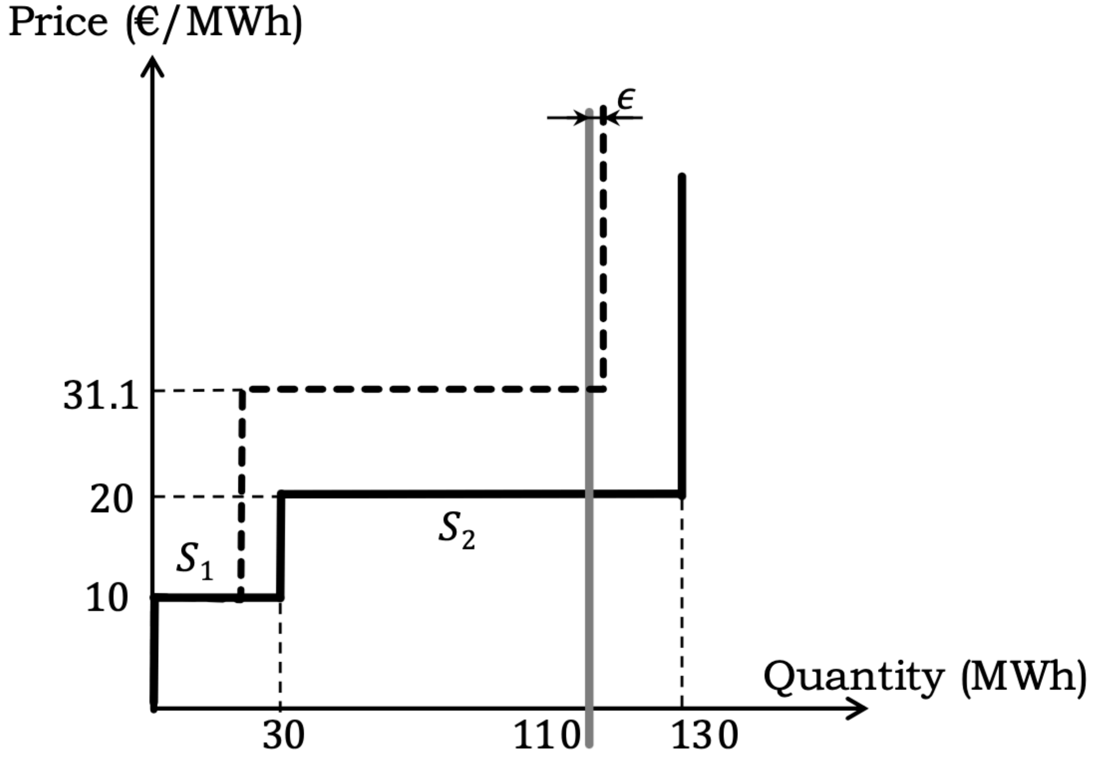

Figure 1: Aggregate marginal cost curve in Example 1.

Definition 3 (Average Incremental Cost Pricing) *The average incremental cost (AIC) prices are the dual variables π^{AIC} associated to the market clearing constraints of the following problem:*

$$z^{AIC} = \min_{c,q,x,f} \sum_{g \in \mathcal{G}} c_g \quad (8a)$$

$$(\pi^{AIC}) \sum_{g \in \mathcal{G}_i} q_{g,t} - D_t^i = \sum_{\substack{l \in \\ from(i)}} f_{l,t} - \sum_{\substack{l \in \\ to(i)}} f_{l,t} \; \forall i \in \mathcal{N}, t \in \mathcal{T} \quad (8b)$$

$$(c, q, x)_g \in \mathcal{X}_g^{AIC} \;\; \forall g \in \mathcal{G} \quad (8c)$$

$$f \in \mathcal{F} \quad (8d)$$

where $\mathcal{X}_g^{AIC}$ is a convex set obtained from $\mathcal{X}_g$ in which each binary variables x_j are relaxed to the continuous interval $0 \leq x_j \leq x_j^*$, where x_j^* is a parameter corresponding to the optimal solution of problem (1); and in which the production is constrained as follows: $0 \leq q_j \leq u_j q_j^* + \epsilon$ where u_j are the commitment (on/off) variables ($u_j \subset x_j$).

Average incremental cost pricing

Main property

Assumption 1. *We assume the model of $\mathcal{X}_g$ is such that $\boldsymbol{0} \in \mathcal{X}_g$ means "inaction" (no production) and if $\boldsymbol{0} \in \mathcal{X}_g$ then $\boldsymbol{0} \in \mathcal{X}_g^{AIC}$.*

Proposition 2 (AIC) *AIC prices ensure zero revenue shortfall for all the suppliers who have possibility of inaction: $RS_g^{gen}(\pi^{AIC}) = 0 \; \forall g \in \mathcal{G} \mid \boldsymbol{0} \in \mathcal{X}_g$.*

AIC prices eliminate the need for discriminatory make-whole payments

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Two auction datasets

*FERC dataset**	*CWE dataset*
~1000 power units	**~70 power units**
Sophisticated unit commitment model	Simpler unit commitment model
Convex & non-convex power units	Only non-convex power units
Possibility of inaction holds	Possibility of inaction does not hold
No network	**Network of 30 bidding zones**
11 load scenarios, 24 periods	12 load scenarios of 24 and 96 periods
Public data	Private data

* Knueven, B., Ostrowski, J., Watson, J.P., 2020. On mixed-integer programming formulations for the unit commitment problem. INFORMS J. Comput. 32, 857–876.
Krall, E., Higgins, M., O'Neill, R.P., 2012. Rto Unit Commitment Test System. Federal Energy Regulatory Commission, p. 98.

AIC results: FERC dataset

Table 1
Results of the FERC dataset (average over 11 scenarios).[a,b]

		MP	CHP	MMWP**	AIC
Dispatch Cost		29,780,000			
Av. Price		28.8	28.7	28.9	29
Suppl. with LOC		3.4%	1.8%	9.5%	6.8%
Av. LOC per Suppl.		628	19	94	570
Δ Consumer Surplus		0%	0%	−0.3%	−0.7%
LOC	Tot.	37,576	323	14,217	48,029
	Conv.	0	67	79	38
	Non-Conv.	37,576	257	14,137	47,991
RS	Tot.	669	19	0	0
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1) AIC prices eliminate the need for make-whole payment

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2) But it also leads to an increase in LOC.
Important asymmetry:
- **Minimizing the LOC (CHP) leads to low RS**
- **Minimizing the RS (AIC) leads to high LOC**

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Proposition 5 (Convex suppliers) *AIC prices do* not *guarantee zero LOC for convex suppliers.*

3) Convex market participants bear some LOC

The burden of non-convexities impact both *convex* (e.g. virtual bids) and *non-convex* suppliers (unlike marginal pricing)

AIC results: CWE dataset

Table 2
Results of the CWE dataset (average over 11 scenarios).[a,b]

		MP	CHP	MMWP**	AIC
Dispatch Cost		5,489,000			
Av. Price		42.4	43.4	52.6	47.2
Suppl. with LOC		35.1%	38.1%	64.2%	52.1%
Av. LOC per Suppl.		3,620	268	26,975	4,244
Δ Consumer Surplus		0%	−1.8%	−17.1%	−13.1%
LOC	Tot.	92,975	8,353	20,765,110	161,312
	Net.	0	1,267	19,513,628	0
	Suppl.	92,975	7,086	1,251,482	161,312
RS	Tot.	13,224	1,887	0	2,087
	Net.	0	0	0	0
	Suppl.	13,224	1,887	0	2,087
	Suppl. PI.	7,286	1,028	0	0
	Suppl. II.	5,937	859	0	2,087

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4) The network tends to exacerbate the differences between pricing schemes: *market is more fragmented thus non-convexities more apparent*

→ **AIC prices 10% higher than MC or CHP**

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Proposition 3 (Suppliers without a possibility of inaction) *A supplier without a possibility of inaction ($\mathbf{0} \notin \mathcal{X}_g$) could bear a revenue shortfall when facing AIC prices.*

5) AIC eliminates MWP, but only for those who have possibility of inaction.

AIC results: CWE dataset

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6) The asymmetry between LOC-minimization and RS-minimization is amplified :

- Minimizing the LOC (CHP) leads to low RS
- Minimizing the RS (AIC) leads to high LOC

→ **Incentives to self-schedule are significantly amplified by AIC pricing.**

AIC results: CWE dataset

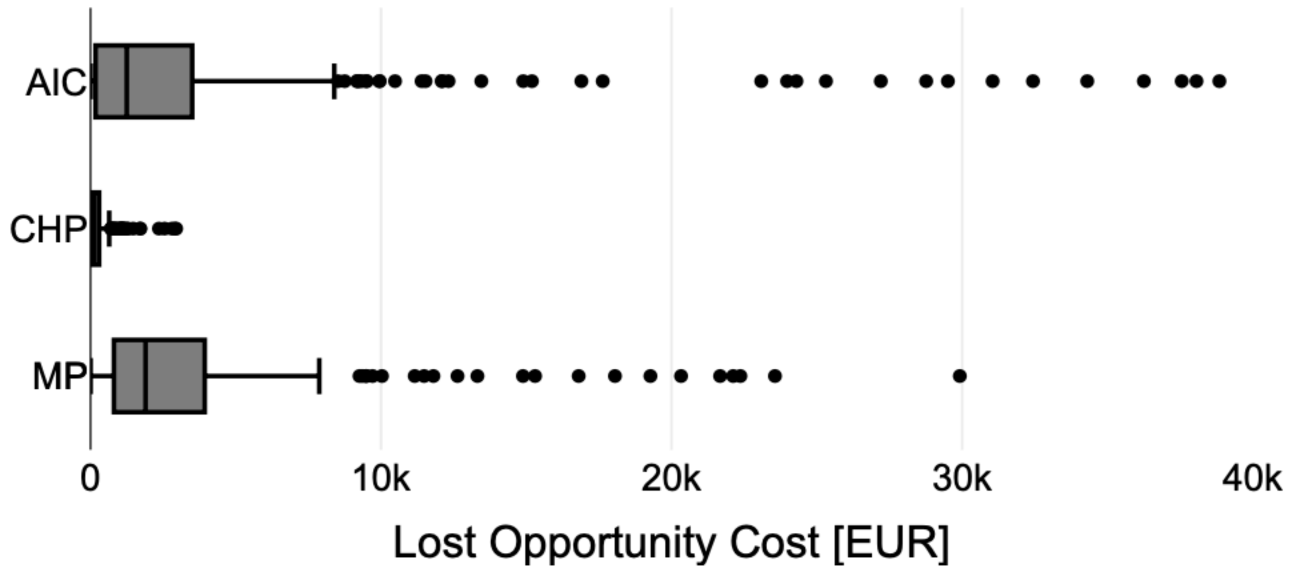

Figure 3: Distribution of LOC among suppliers (CWE cases).

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7) Because it leads to higher prices, AIC leads to less consumer surplus in the short-term

AIC results: CWE dataset

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Proposition 4 (Network) *AIC prices ensure zero LOC for the network. If $\mathbf{0} \in \mathcal{F}$ (i.e. $f = 0$ is feasible), this also implies zero RS for the network.*

8) AIC price leads to zero LOC for the network → there is no arbitrage in the network

AIC results: CWE dataset

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9) Compared to alternative methods that minimize RS, AIC pricing leads much smaller LOC

Content

Conclusions

- AIC pricing ensures zero RS for suppliers, thus **eliminating the need of make-whole payments** (Proposition 2)
- But only for suppliers who have **possibility of inaction** (Proposition 3).
- It **eliminates arbitrage opportunities in the network** (zero "network LOC", Proposition 4).
- Fully eliminating the RS by means of the uniform price signal, however, **can increase the LOC significantly**, thus creating the risk of exacerbating self-scheduling behaviour.
- Since it leads to higher uniform prices, the AIC price tends to **lower short-term consumer surplus** (although this might also increase investment incentives).
- AIC prices can be **sensitive to formulation choices** (Proposition 6).

Thank you!

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