Does Financial Trading Smooth Non-Convex Markets?

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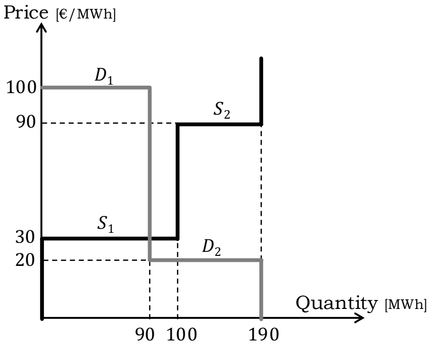
Markets with non-convexities

- In a **non-convex market**, a Walrasian equilibrium is not guaranteed to exist
- Some "asymptotic" positive results:
  - Quasi-equilibrium & **market size effect** (Starr, 1969)
  - Equilibrium existence with a continuum of traders (Aumann, 1964)
- Yet non-convexity remains a problem in real-world auctions (Milgrom, 2017).
- **Electricity wholesale markets** are typically organized as a sealed-bid auction with uniform pricing which includes non-convex bids (Wilson, 2002; Cramton, 2017):
  - Complex cost and constraints of production modelled into the auction
  - Unit commitment model in the US (Knueven et al., 2020)
  - Block orders in the EU market
- Main implication: equilibrium is not guaranteed to exist although it might exist in some cases (Bikhchandani and Mamer, 1997).

Markets with non-convexities

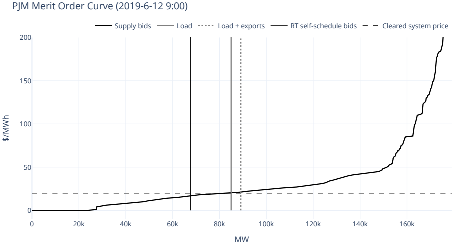


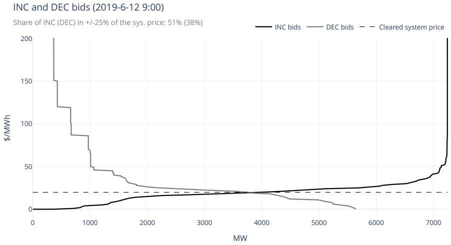
- S_1 is a non-convex: all or nothing
- the Walrasian auctioneer maximizes the total surplus
- clears S_1, D_1, and a share of D_2 (i.e.10MWh).
- no uniform price supports the cleared allocation

Markets with non-convexities

- The auctioneer complements the uniform price with **side payments** to recover an equilibrium (O'Neill et al., 2005).
- These "out-of-market" payments are undesirable: discriminatory, non transparent, induce gaming, affect investment signal (Byers and Hug, 2023).
- They imply ***relevant cost information is not reflected in the price signal***
- Broad literature on **designing pricing mechanisms dealing with non-convexities** (Milgrom and Watt, 2025), specifically for electricity markets (O'Neill et al., 2005; Gribik et al., 2007; Chao, 2019; Stevens et al., 2024; Ahunbay et al., 2025)

Financial trading

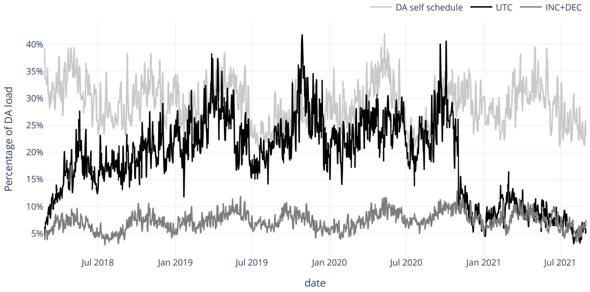




- Electricity markets typically organised as **sequential auctions**:
  - day-ahead (forward) market
  - real-time (spot) market
- Multi-period market (24 hours), multi-products (thousands of nodes)
- Different types of bids in the market:
  - **physical bidders**: dispatchable generation (***non-convex*** bids) and self-schedule generation (convex bids)
  - **virtual – financial – bidders** (***convex*** bids): INC, DEC and UTC

Financial trading

Electricity auctions such as PJM include large volumes of virtual (financial) bids



Average hourly volumes	GW
Day-ahead load	88
Physical bids:	
– Non-convex dispatch. bids	139
– DA self-schedule bids	25
Virtual bids:	
– Cleared INC bids	2.5
– Cleared DEC bids	3.8
– Cleared UTC bids	15.4

Contributions

The benefits of financial trading had been extensively studied in the literature on electricity markets (Hogan, 2016), such as:

- market power mitigation (Ito and Reguant, 2016; Mercadal, 2022)
- price convergence and cost savings (Jha and Wolak, 2023)

$\Rightarrow$ We investigate whether financial traders induce a ***smoothing effect*** in non-convex markets ($\neq$ **market-size effect**)

> *"Including virtual transactions [...] would have the effect of smoothing the day-ahead commitment and dispatch problem and reducing the required uplift [side] payments." (Hogan, 2016)*

Our paper makes two main contributions:

- **Model** formalizing the ***smoothing effect***
- **Empirical analysis** on PJM Interconnection data

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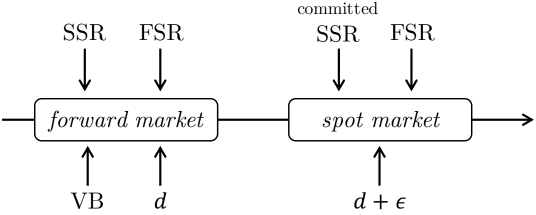
Model environment

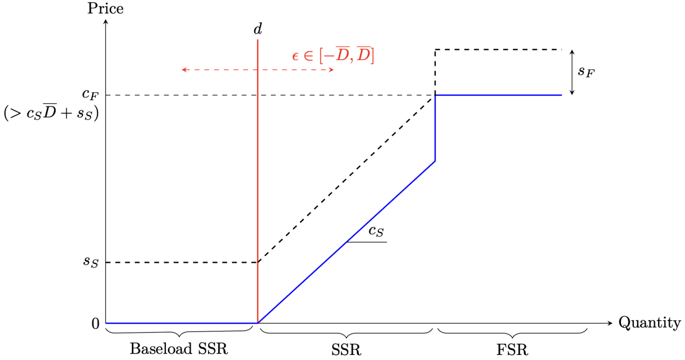
Two-stages market: forward day-ahead (DA) & spot real-time (RT) markets



Market participants:

- Exogenous inelastic demand: d (in DA) + random shock $\epsilon \in [-\overline{D}, \overline{D}]$ (in RT)
- Physical bidders: slow-start resources (SSR) and fast-start resources (FSR)
- Virtual bidders: arbitrager between DA and RT

Model environment



- fixed (non-convex) start-up cost s and a variable cost of production
- slow-start resources (SSR): $C_S(q) = \frac{c_S}{2}q^2 + s_S q$
- fast-star resources (FSR): $C_F(q) = c_F q + s_F q$
- we assume $c_F > c_S\overline{D} + s_S$ and $s_F > s_S$
- physical bidders bid truthfully in the sequential markets
- auctioneer: welfare-max allocation & marginal pricing + side payments

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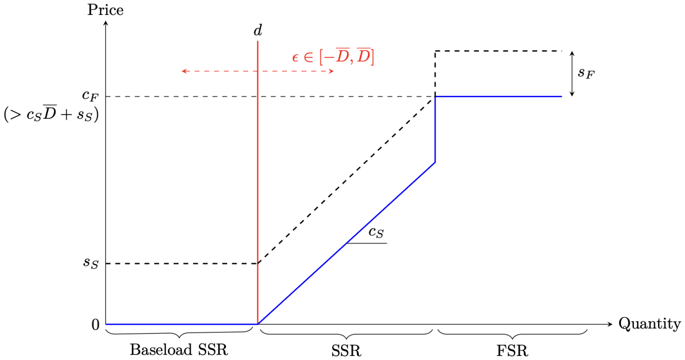
Equilibrium **without** virtual trading



Equilibrium prices, side payments and cost

$$p_{DA} = 0, \qquad SP_{DA} = s_S d \tag{1}$$

$$\mathbb{E}_\epsilon(p_{RT,\epsilon}) = \frac{c_F}{2}, \qquad \mathbb{E}_\epsilon(SP_{RT,\epsilon}) = \frac{s_F\overline{D}}{4} \tag{2}$$

$$Cost = s_S d + \frac{\overline{D}}{4}(c_F + s_F) \tag{3}$$

Equilibrium **with** virtual trading

Virtual bidders competing *à la Bertrand* until no-arbitrage condition holds: $p_{DA} = \mathbb{E}_\epsilon(p_{RT,\epsilon}) \Rightarrow$ activation of k slow-start units

Equilibrium prices

$$p_{DA} = P = c_S k + s_S \tag{4}$$

$$\mathbb{E}_\epsilon(p_{RT,\epsilon}) = \frac{1}{2\overline{D}} \left(\int_0^k c_S \epsilon d\epsilon + \int_k^{\overline{D}} c_F d\epsilon \right) = \frac{1}{2\overline{D}} \left(\frac{c_S}{2} k^2 - c_F k + c_F \overline{D} \right) \tag{5}$$

with

$$k^* = \frac{c_F + 2c_S\overline{D} - \sqrt{c_F^2 + (2c_S\overline{D})^2 + 2c_F c_S \overline{D} + 4c_S s_S \overline{D}}}{c_S} \tag{6}$$

Proposition

Under perfect virtual trading, k^* SSR are committed in DA, such that (focusing on $k^* > 0$):

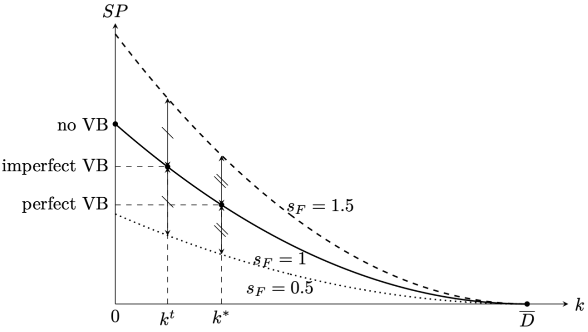
1. $\partial k^*/\partial s_S < 0$: *the higher the fixed cost of slow-start units, the lower k^*.*
2. $\partial k^*/\partial c_F > 0$: *the higher the variable cost of fast-start units, the higher k^*.*
3. $\partial k^*/\partial c_S < 0$: *the higher the variable cost of slow-start units, the lower k^*.*
4. $\partial k^*/\partial s_F = 0$: *$k^*$ is insensitive to s_F.*

Equilibrium **with** virtual trading: side payments

Equilibrium side payments

$$SP_{DA} = 0 \tag{7}$$

$$\mathbb{E}_\epsilon(SP_{RT,\epsilon}) = \frac{1}{2\overline{D}} \int_k^{\overline{D}} s_F(\epsilon - k)d\epsilon = \frac{s_F\overline{D}}{4} - \frac{s_F k}{2}\left(1 - \frac{k}{2\overline{D}}\right) \tag{8}$$



Proposition

Perfect virtual trading decreases expected side payments both in day-ahead and in real-time.

Proposition

Real-time side payments are linearly increasing with s_F.

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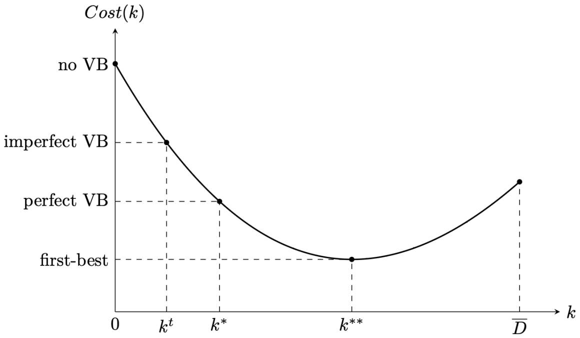
Equilibrium **with** virtual trading: efficiency

Equilibrium cost

$$Cost(k) = \underbrace{s_S d + \frac{\overline{D}}{4}(c_F + s_F)}_{\text{cost without VB}} + \underbrace{s_S k}_{\text{sunk cost}} \underbrace{- \frac{k^3 c_S}{6\overline{D}} + \frac{k^2(c_F + s_F)}{4\overline{D}} + \frac{k^2 c_S}{4} - k\frac{(c_F + s_F)}{2}}_{\text{expected cost savings}} \qquad (9)$$



Proposition (Efficiency 1)

Perfect virtual trading improves cost efficiency compare to no virtual trading.

Proposition (Efficiency 2)

Perfect virtual trading does not reproduce the first-best efficient day-ahead commitment plan k^{**}: it entails lower commitments of SSR than optimal, thus $k^* < k^{**}$.

Equilibrium **with** virtual trading: transaction fee

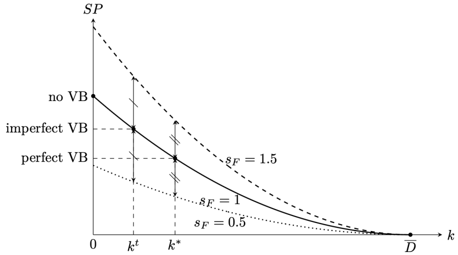
Virtual traders competing *à la Bertrand* now leads to the following arbitrage condition:

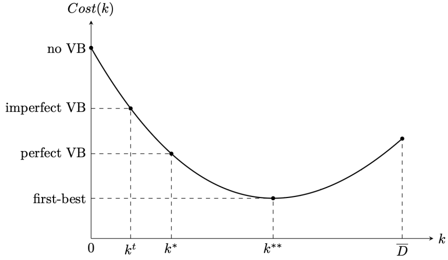
$$\mathbb{E}_\epsilon(p_{RT,\epsilon}) - p_{DA} = t$$

Proposition (VB with transaction cost t)

Virtual trading with transaction cost t leads to $k^t \le k^*$ such that:

- $Cost(k^*) \le Cost(k^t) \le Cost(0)$
- $SP(k^*) \le SP(k^t) \le SP(0)$





Main takeaways of the model

1. **Price convergence**: virtual trading improves price convergence between DA and RT, although a transaction fee t would entail a persistent price difference
2. **Efficiency**: virtual trading improves DA commitment decisions (more activations of SSR) which lowers the expected costs (cf. Jha and Wolak (2023))
3. **Side payments**: virtual trading mitigates the side-payments. *High side payments means some cost information is not reflected in the market price and is instead handled by out-of-market payments.*
   - DA: virtual trading leads to prices that endogenize SSR fixed costs which reduces the day-ahead side payments.
   - RT: by anticipating the risk of FSR activation, the likelihood of lumpy FSR activations, associated with large side-payments, is reduced.

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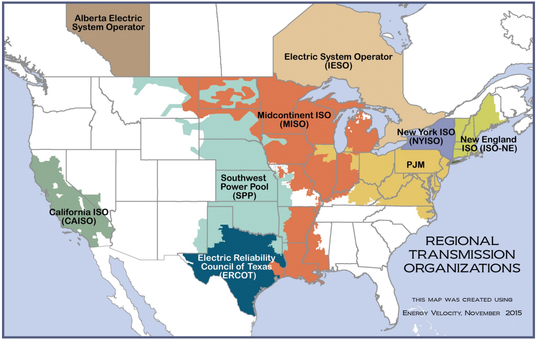
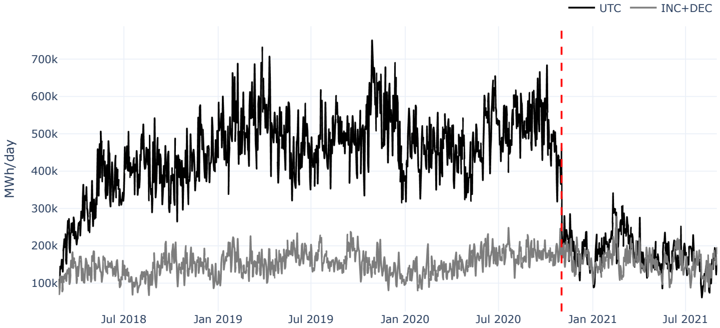
PJM Interconnection



- US largest electricity market
- 67 million people across 13 states
- $\geq$1,400 power plants ($\sim$180 GW)
- Power grid with thousands of nodes

Transaction fee on virtual bids

On November 1, 2020, a transaction fee was introduced on virtual bids (UTC). As a consequence, the volume of virtual bids (UTC) significantly dropped.



"We find that PJM's current uplift allocation rules are ***unjust, unreasonable, and unduly preferential*** *because they do not allocate uplift to UTCs. Accordingly, we direct PJM to submit [...] a replacement rate that revises PJM's current uplift allocation rules to allocate uplift to UTCs in a manner that treats a UTC, for uplift allocation purposes, as if the UTC were equivalent to a DEC at the sink point of the UTC. As a result, under the replacement rate, UTCs will be allocated both real-time uplift and day-ahead uplift." (FERC, 2020)*

Transaction fee on virtual bids

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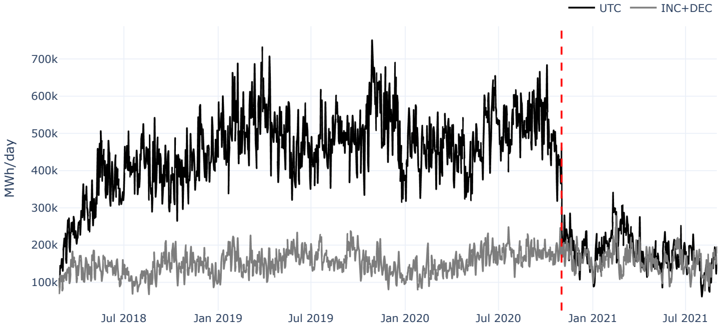
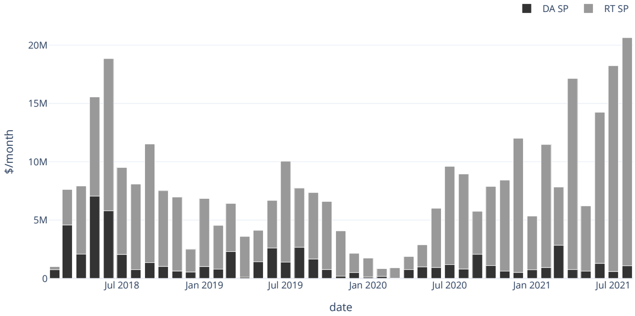


⇒ This is aligned with expectations and with our model: a transaction fee leads to a volume reduction

⇒ Our model predicts that the decline in financial trading volume should coincide with an increase in side payments

Empirical strategy

- We leverage extensive market data from PJM Interconnection (bid data, virtual bid data, prices, congestions, outages, etc.)
- We focus on the period:
  - from February 20, 2018 (change of virtual products definition)
  - to September 1, 2021 (change of auction pricing rule)
- To estimate the impact of the policy, we estimate **two models**: (1) **DA side payments** and (2) **RT side payments** (most SP are in RT)



Empirical strategy: RT model

$$SP_{RT,t} = \alpha + \beta \text{Treatment}_t + \gamma_{fsr} s_{F,t} + \gamma_{shock} loadShock_t + \eta \mathbf{X}_t + u_t$$

- $SP_{RT,t}$: real-time side payments for each day t.
- Treatment_t: transaction fee introduced on November 1, 2020
  - $\beta =$ treatment effect
  - We expect $\beta > 0$.
- $s_{F,t}$: FSR fixed costs $\Rightarrow$ we expect $\gamma_{fsr} > 0$
- $loadShock_t$: load shocks $\Rightarrow$ we expect $\gamma_{shock} > 0$
- $\mathbf{X}_t$: control variables $\Rightarrow$ PJM provides a list of the main drivers of side payments (PJM, 2026):
  - Load
  - Grid congestions
  - Market prices, gas prices
  - Generation availability
  - Self-scheduling
  - Emergency procedure events

Empirical strategy: DA model

- Two main differences between DA and RT side payments
  - they are substantially smaller in magnitude
  - they are equal to zero on a significant share of days
- somewhat consistent with the prediction of our model: participation of virtual trading can fully eliminate day-ahead side payments.

	Day-ahead			Real-time		
	All Days	Pre-Treat.	Post-Treat.	All Days	Pre-Treat.	Post-Treat.
Mean	47,216	51,358	33,606	216,096	167,675	375,218
Median	7367	7307	7602	109,933	91,245	220,091
Stand. Dev.	86,404	92,022	62,822	296,809	210,648	446,403
Max	958,694	958,694	646,474	2,604,937	1,789,976	2,604,937
% of 0	22%	24%	16%	0.08%	0.1%	0%
# observations	1273	976	297	1273	976	297

Table: Day-ahead and real-time side-payments [$/day], pre- and post-treatment.

Empirical strategy: DA model

- Two main differences between DA and RT side payments
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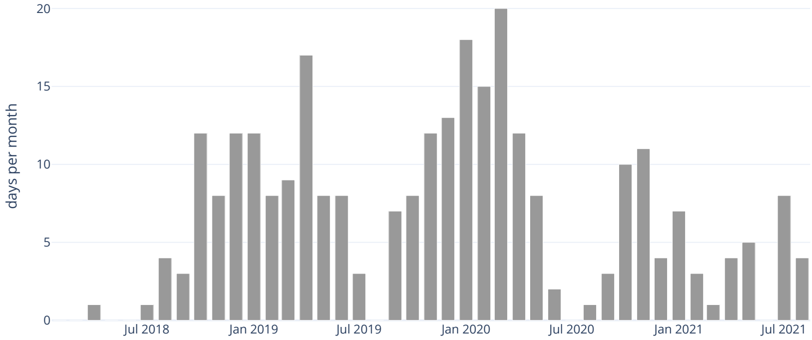

Empirical strategy: DA model

- Two main differences between DA and RT side payments
  - they are substantially smaller in magnitude
  - they are equal to zero on a significant share of days
- somewhat consistent with the prediction of our model: participation of virtual trading can fully eliminate day-ahead side payments.
- we estimate the following binary response **logit model**:

$$\mathbb{P}(SP_{DA,t} > 0|x) = G(\alpha + \beta \text{Treatment}_t + \eta \mathbf{X}_t)$$

  - where G is the logistic function $(G(z) = e^z/(1+e^z))$
  - $SP_{DA,t}$ = day-ahead side payments at date t
  - $\mathbb{P}(SP_{DA,t} > 0|x)$ = probability of having positive side payments
  - Treatment_t = indicator representing the introduction of the transaction fee $\Rightarrow$ we expect $\beta > 0$
  - $\mathbf{X}_t$ = control variables.

Main results: RT model

	(I)	(II)	(III)	(IV)	(V)	(VI)
Treatment Effect	135.09***	159.79***	160.67***	207.78***	127.44***	108.75***
(in K dollars)	(16.55)	(16.24)	(16.44)	(17.93)	(17.13)	(16.77)
FSR fixed cost	33.97**	11.4	6.51	31.59***	42.38**	13.3
	(12.64)	(8.63)	(8.71)	(9.57)	(13.29)	(10.49)
DA-RT load error	0.0003***	0.0003***	0.0003***	0.0005***	0.0003***	0.0003***
	(3.7e-05)	(3.7e-05)	(3.7e-05)	(4e-05)	(3.9e-05)	(3.7e-05)
System stress controls	X				X	X
Date controls	X	X			X	X
Market controls	X	X	X		X	X
Sample size	1276	1276	1276	1276	1216	1395
R squared	0.46	0.42	0.41	0.21	0.46	0.41

Table: Treatment effect on Real-Time Side Payments

Main results: RT model

Four main observations:

- The average treatment effect is positive and stand for an increase of 135k$ (with a 95% confidence interval of 103 to 168k$).
    - Statistical significance and robust to add/removal of covariables that we add + time window robustness check.
- From Table 1, pre-treatment magnitude of the side payments was 167k$. Thus the introduction of the the transaction fee increased by roughly 80% the real-time side payments.
- As expected, the FSR fixed costs and the load shock both increase the side payments.
- The fact that the effect is slightly smaller in column (VI) is aligned with what we expect, since the implementation of fast-start pricing should have mitigated the side payments.

Main results: DA model

	(I)	(II)	(III)	(IV)	(V)	(VI)
Logit Coefficient	0.6954^{***}	0.7156^{***}	0.5416^{**}	0.5189^{**}	0.7840^{***}	0.7232^{***}
	(0.1996)	(0.1923)	(0.1870)	(0.1757)	(0.2005)	(0.1880)
System stress controls	X				X	X
Date controls	X	X			X	X
Market controls	X	X	X		X	X
Sample size	1276	1276	1276	1276	1216	1395

Table: Treatment effect on probability of having Day-Ahead Side Payments (logit model)

The logit coefficient (0.7) means the treatment double the odds ($e^{0.7} \approx 2.02$) of non-zero DA side-payments.

Given an average probability of non-zero DA side payments before treatment of 76%, it means the treatment increased on average by $+10\%$ the chances of having non-zero side-payments in day-ahead.

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Stakeholder viewpoints on virtual trading

- PJM (2015) argued that virtual bids may give rise to side payments by distorting commitments ⇒ virtual bids should be charged a transaction fee
  - PJM argument not based on an equilibrium analysis,
  - stylized examples assuming exogenous bidding behaviour for traders.
- Hogan (2016) argued against it as this would deter participation and affect market efficiency.
- In 2020, FERC order on UTC transaction fee gave rise to contrasted views:
  - The utilities were in favor of this policy

  "PJM Utilities Coalition conclude that because UTCs are transacting in the markets, they should receive their respective share of the costs that result from market operations." (FERC, 2020)

  - In contrast, some trading firms active in PJM market argued that

  "because UTCs promote price convergence and help with price formation, allocation of uplift to UTCs disincentivizes UTCs' efficiency enhancing characteristics" (FERC, 2020)

- FERC justification: "unjust, unreasonable, and unduly preferential" that INC and DEC pay a fee while UTC don't

Summary of our findings

Our article argues that the introduction of financial traders induces a ***smoothing effect***. Financial traders are arbitragers that bid close to the market price, resulting into the addition of a large volume of convex bids close to the margin.

- **Model**: financial trading mitigates the side payments needed to sustain an equilibrium.
  - DA: virtual traders set the prices and internalize fixed costs ⇒ fixed costs reflected in the uniform price ⇒ no side payments.
  - RT: higher price convergence and better commitment decisions in DA ⇒ less RT lumpy activations of FSR associated with side payments.
- **Empirical analysis**: the introduction of a transaction fee on financial traders in PJM, on November 1, 2020, led to a substantial decline in financial trading volume which led to an increase of side payments.
  - DA: the likelihood of non-zero side-payments increased by 10%
  - RT: the amount of side-payments increased by 80%

⇒ virtual trading reduces side payments, a transaction fee is counter-productive

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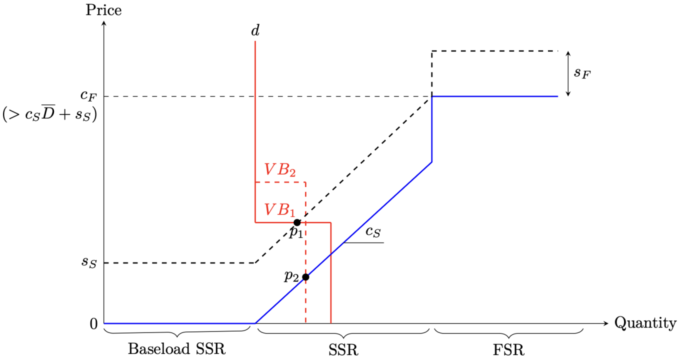
Pricing mechanism



- *Allocation rule*: the auctioneer clears the allocation that maximizes social welfare
- *Pricing and settlement rule*: the auctioneer uses marginal pricing rule + producers that do not break even receive side payments to offset their losses.

Numerical illustration

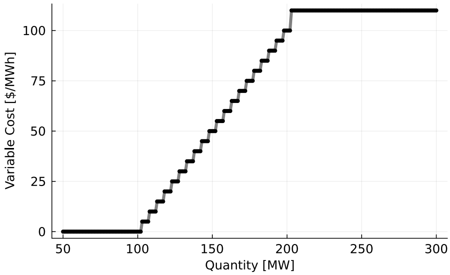
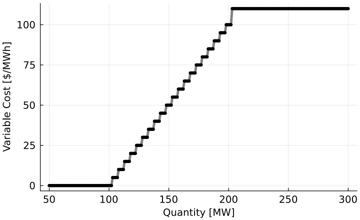


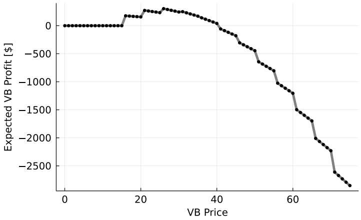
Figure: Merit order curve

- 20 SSR and 20 FSR, capacity of 5MW, min production limit of 2 MW.
- SSR: variable cost of $i \times 5\$/\text{MWh}$ for $i \in \{1, 20\}$, fixed cost of $50
- baseload SSR: 100MW with variable cost 0 and fixed cost of $500.
- FSR: variable cost of $110\$/\text{MWh}$, fixed cost of $75
- $d = 100\text{MW}$, shocks ϵ uniformly distributed in $\{2i | i \in -50 \ldots 50\}$.

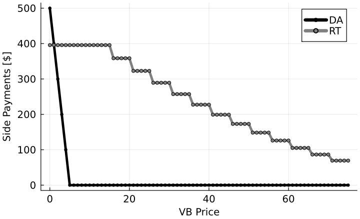
Numerical illustration



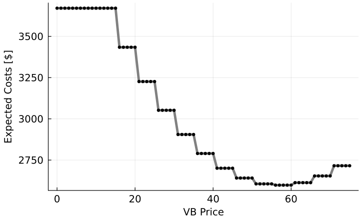
(a) Merit order curve



(b) Virtual bidders profit



(c) Side Payments



(d) Total expected costs

Markets with non-convexities

$$z^* = \min_{c,q,x} \sum_{g \in \mathcal{G}} c_g \tag{10a}$$

$$\sum_{g \in \mathcal{G}} q_{g,t} = D_t \qquad \forall t \in \mathcal{T} \tag{10b}$$

$$(c, q, x)_g \in \mathcal{X}_g \qquad \forall g \in \mathcal{G} \tag{10c}$$

- Most **electricity markets** are organized as a sealed-bid auction with uniform pricing which includes non-convex bids (Wilson, 2002; Cramton, 2017)
- $\mathcal{X}_g$ may be **non-convex** (unit commitment model (Knueven et al., 2020))
- With non-convexities, a **Walrasian equilibrium is not guaranteed to exist** (strong duality does not hold).

Markets with non-convexities

Definition (Walrasian Equilibrium)

Allocation (c^*, q^*, x^*) with price π constitute a Walrasian equilibrium if

1. for each supplier g, $(c^*, q^*, x^*)_g$ optimizes the profit problem under price π
2. the market clears (constraint (10b)).

Proposition (Equilibrium existence condition)

There exists a price π^E such that $(\pi^E, (c^*, q^*, x^*))$ is a Walrasian equilibrium in model (10) *if and only if (c^*, q^*, x^*) is a solution to the convex relaxation of problem* (10) *in which $\mathcal{X}_g$ are replaced by $\mathrm{conv}(\mathcal{X}_g)$.*

Markets with non-convexities: the *market-size effect*

Market Size			Convex Hull Pricing		Marginal Pricing	
Number of Plants	Av. Hourly Load (MW)	Tot. Cost ($)	LOC ($)	LOC (% Tot. Cost)	LOC ($)	LOC (% Tot. Cost)
50	4,900	1,820,308	11,222	0.62%	276,383	15.18%
100	9,800	3,631,286	13,114	0.36%	538,713	14.84%
200	19,600	7,245,546	9,202	0.13%	1,060,574	14.64%
300	29,400	10,857,007	2,492	0.02%	1,579,297	14.55%
400	39,200	14,475,824	3,136	0.02%	2,105,629	14.55%
500	49,000	18,099,571	8,711	0.05%	2,636,708	14.57%
1000	98,000	36,183,999	2,280	0.01%	5,258,840	14.53%

Proposition (LOC Bound 1)

Under CHP, the total side payments are bounded. The bound depends on the shape of $\mathcal{X}_g$, but is independent of $|\mathcal{G}|$: $\lim_{|\mathcal{G}|\to\infty} LOC(\pi) < \Gamma$.

Proposition (LOC Bound 2)

Under MP, the total side payments are not necessarily bounded: it could be that $\lim_{|\mathcal{G}|\to\infty} LOC(\pi) \to \infty$.

Equilibrium **with** virtual trading: side payments

Some remarks:

- **RT**: real-time side payments not fully eliminated by virtual trading
  - One way to improve these issues is by amending the **pricing rule**, deviating from marginal pricing

    (e.g. setting the price to $c_F + s_F$ instead of c_F when FSR are activated, cf. "fast-start pricing approaches").
  - Our goal is to highlight how, besides the design of pricing rules, virtual trading contributes to mitigating the issues implied by non-convexities.
- **DA**: virtual trading fully eliminates DA side payments. Some caveats:
  - Financial trading volume: our model assumes an **infinite quantity of virtual traders**, bidding at a price $P \Rightarrow$ DA price set by P. A limited volume could lead to cases where a SSR set the price, which would entail side-payments.
  - Neglected complexities: our model neglects the multi-period nature of the market, inter-temporal constraints and most unit commitment constraints that are encountered in actual markets.

Empirical strategy: RT model

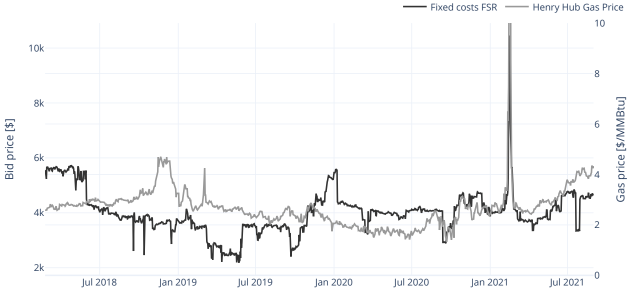
$$SP_{RT,t} = \alpha + \beta \text{Treatment}_t + \gamma_{fsr} s_{F,t} + \gamma_{shock} loadShock_t + \eta \mathbf{X}_t + u_t$$

- $SP_{RT,t}$: real-time side payments for each day t.
- Treatment_t: transaction fee introduced on November 1, 2020
  - $\beta =$ treatment effect: by how much did the transaction fee increased the amount of real-time side payments.
  - We expect $\beta > 0$.
- $s_{F,t}$: FSR fixed costs
- $loadShock_t$: load shocks
- $\mathbf{X}_t$: control variables

Empirical strategy: RT model

$$SP_{RT,t} = \alpha + \beta \text{Treatment}_t + \gamma_{fsr} s_{F,t} + \gamma_{shock} loadShock_t + \eta \mathbf{X}_t + u_t$$

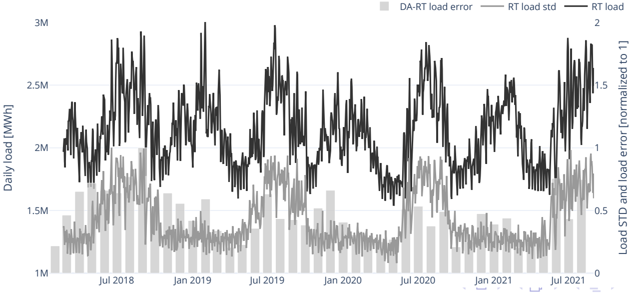
- $SP_{RT,t}$: real-time side payments for each day t.
- Treatment_t: indicator variable representing the introduction of a transaction fee on virtual bids on November 1, 2020
- $s_{F,t}$: **FSR fixed costs** $\Rightarrow$ we expect $\gamma_{fsr} > 0$
- $loadShock_t$: load shocks
- $\mathbf{X}_t$: control variables



Empirical strategy: RT model

$$SP_{RT,t} = \alpha + \beta \text{Treatment}_t + \gamma_{fsr} s_{F,t} + \gamma_{shock} loadShock_t + \eta \mathbf{X}_t + u_t$$

- $SP_{RT,t}$: real-time side payments for each day t.
- Treatment_t: indicator variable representing the introduction of a transaction fee on virtual bids on November 1, 2020
- $s_{F,t}$: FSR fixed costs
- $loadShock_t$: **load shocks** $\Rightarrow$ we expect $\gamma_{shock} > 0$
- $\mathbf{X}_t$: control variables



Empirical strategy: RT model

$$SP_{RT,t} = \alpha + \beta \text{Treatment}_t + \gamma_{fsr} s_{F,t} + \gamma_{shock} loadShock_t + \eta \mathbf{X}_t + u_t$$

- $SP_{RT,t}$: real-time side payments for each day t.
- Treatment_t: indicator variable representing the introduction of a transaction fee on virtual bids on November 1, 2020
- $s_{F,t}$: FSR fixed costs
- $loadShock_t$: load shocks
- $\mathbf{X}_t$: **control variables** $\Rightarrow$ controlling for additional market complexities (multi-period market, grid constraints, etc.). PJM provides a list of the main drivers of side payments (PJM, 2026):
    - Load
    - Grid congestions
    - Market prices + gas prices
    - Generation availability
    - Self-scheduling
    - Emergency procedure events