Application of the Level Method for Computing Locational Convex Hull Prices

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CORE | Louvain Institute of Data Analysis and Modeling in economics and statistics

Pricing under non-convexities

$$\min_{c,p,u,f} \sum_{g\in\mathcal{G}} c_g \tag{1a}$$

$$(\pi_t^i) \quad \sum_{g\in\mathcal{G}_i} p_{g,t} - D_t^i = \sum_{\substack{l\in\\ from(i)}} f_{l,t} - \sum_{\substack{l\in\\ to(i)}} f_{l,t} \quad \forall i,t \tag{1b}$$

$$(c_g, p_{g,t}, u_{g,t}) \in \mathcal{X}_g \quad \forall g\in\mathcal{G} \tag{1c}$$

$$f \in \mathcal{F} \tag{1d}$$

Typical market clearing problem

- Minimize costs
- Market clearing constraint (supply = demand)
- Supply (generator / orders) technical constraints
- Network constraints

- In a competitive market, a **market operator** computes:
  - Market **matches** (commitment & dispatch)
  - Market **prices**

$$\max_{c,p,u} \sum_t p_{g,t}\pi_t^{i(g)} - c_g \tag{2a}$$

$$(c_g, p_{g,t}, u_{g,t}) \in \mathcal{X}_g \tag{2b}$$

- Good price signals are essential in a competitive market

- In **convex cases**, the dual variable associated to (1b) is an **equilibrium price**

Pricing under non-convexities

$$\min_{c,p,u,f} \sum_{g\in\mathcal{G}} c_g \quad (1a)$$

$$(\pi_t^i) \quad \sum_{g\in\mathcal{G}_i} p_{g,t} - D_t^i = \sum_{\substack{l\in\\ from(i)}} f_{l,t} - \sum_{\substack{l\in\\ to(i)}} f_{l,t} \quad \forall i,t \quad (1b)$$

$$(c_g, p_{g,t}, u_{g,t}) \in \mathcal{X}_g \quad \forall g \in \mathcal{G} \quad (1c)$$

$$f \in \mathcal{F} \quad (1d)$$

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$$c_g \quad (2a)$$

$$\in \mathcal{X}_g \quad (2b)$$

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Non-convexities are at the heart of power system operations:
- the **Network model** (the AC power flow equations)
- **Market orders** (the MIP constraints that describe the market offers)

Convex hull pricing (CHP) minimizes uplifts

- Under non-convexity: impossible to find a set of equilibrium uniform prices

- Combine the **uniform price** with **discriminatory payments** (**uplifts**) to restore the proper incentives

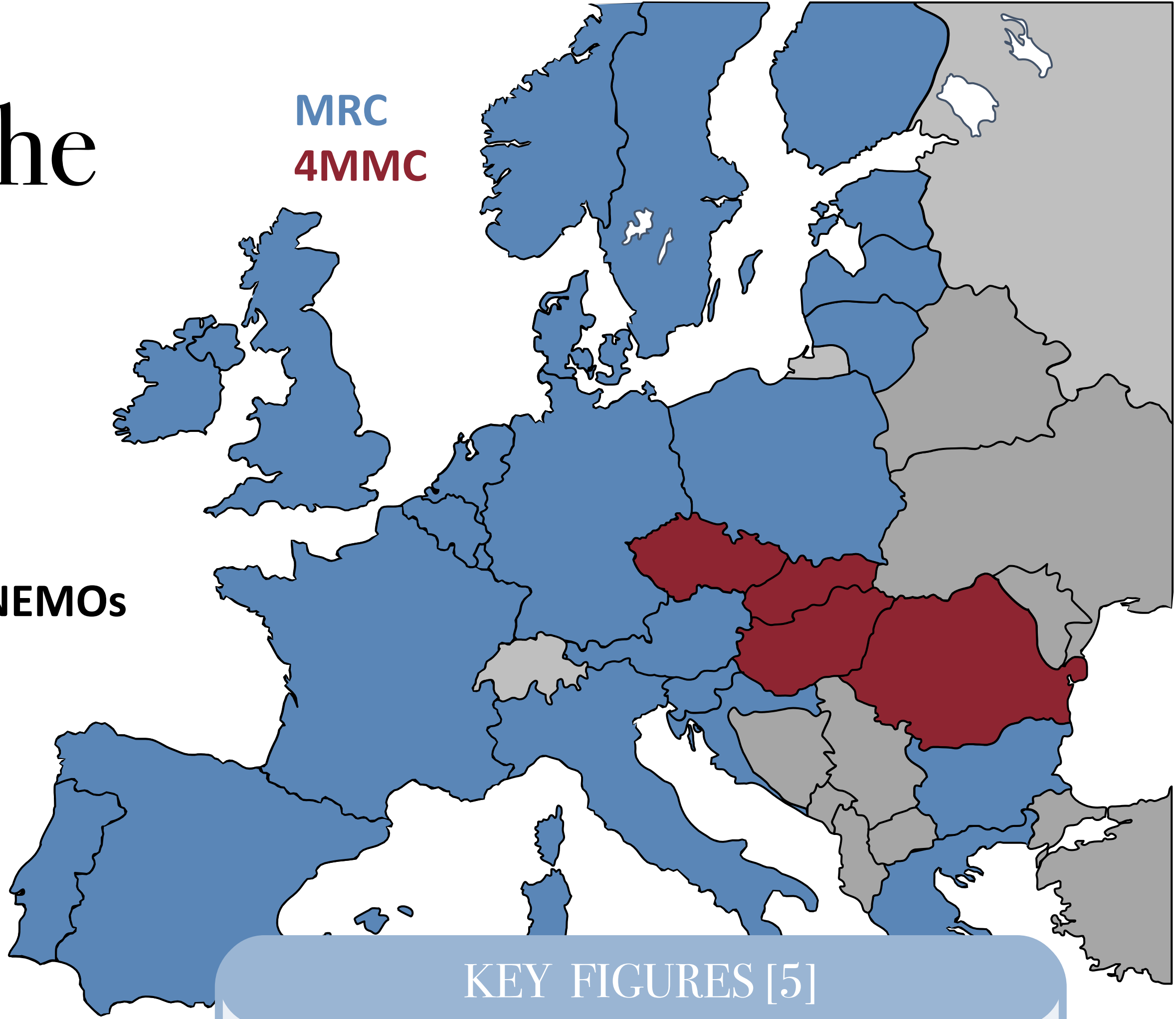
- Popular non-uniform pricing schemes include:
  - **IP pricing** proposed by O'Neill [1]
  - **Convex hull pricing** proposed by Gribik and Hogan [2-3]
  - **Extended LMP pricing**, applied in the PJM market [4]

- **Convex Hull Pricing minimize uplifts**
- Uplifts are undesirable
  - Distort the incentives of bidders
  - Create revenue adequacy problems for the market operator
- But CHP is computationally challenging!

→ **Our research aims at addressing these computational challenges**

CHP is also considered in the EU day-ahead market

- CHP has been so far mainly debated in the US (e.g. PJM)
- But **it has recently received consideration by the European NEMOs** as a possible option for the European DA energy auction [5]
- **The European DA market** is a huge market
  - Runs once per day
  - Couples EU countries
  - Set the **dispatch** and the **electricity prices**
  - Complex institutional structure:
    - **TSO** (regulated monopoly): Elia, RTE, Statnett, etc.
    - **NEMO**: EPEX SPOT, Nord Pool, etc.
  - Market clearing **algorithm: EUPHEMIA**



KEY FIGURES [5]

- Geographical ext.: **27** countries
- TSOs: **35**
- NEMOs: **16**
- Traded vol.: **4.08** TWh/session
- Welfare: **8.8** B€/session
- Convex bids: **>200,000** /session
- Non-convex bids: **>5,000** /session

Plan

Introduction	• Pricing under non-convexity • Main concepts related to Convex Hull Pricing (CHP) • EU day-ahead market context
CHP formulation	• Mathematical formulation of CHP • Algorithmic schemes to compute CHP
The Level Method	• Kelley's algorithm • Level stabilization • Adaptation of the basic algorithm to CHP specificities
Numerical Results	• Size of the EU day-ahead market problem • Comparison of the Level Method and the Dantzig Wolfe algorithm on EU instances
Conclusion	• Perspective for future research

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Minimizing the uplifts amounts to solving a Lagrangian relaxation

$$\pi^{CHP} = \arg\max_{\pi} L(\pi) \tag{3}$$

$$L(\pi) = \sum_{i,t} \pi_t^i D_t^i \tag{4a}$$

$$- \sum_{g \in \mathcal{G}} \max_{(c,p,u)_g \in \mathcal{X}_g} \left\{ \sum_t p_{g,t} \pi_t^{i(g)} - c_g \right\} \tag{4b}$$

$$+ \min_{f \in \mathcal{F}} \left\{ \sum_{i,t} \pi_t^i \left(\sum_{\substack{l \in \\ from(i)}} f_{l,t} - \sum_{l \in to(i)} f_{l,t} \right) \right\} \tag{4c}$$

Property 1 (Concave).
Function $L(\pi)$ is concave in π.

Property 2 (Non-smooth).
Function $L(\pi)$ is a *non-smooth* (*piecewise linear*) function, i.e. each facet can be seen as corresponding to a set of binary (commitment) decisions u_g.

Property 3 (First-order oracle).
A first-order oracle is available, i.e. given a price π, both the function value $L(\pi)$ as well as its supergradient g $\in \partial L(\pi)$ can be evaluated.

Property 4 (Supergradient).
Let (c^*, p^*, u^*, f^*) be the optimal reactions to π (solving respectively (4b) and (4c)). Then

$$g = D_t^i - \sum_{g \in \mathcal{G}} p_{g,t}^* + \sum_{l \in from(i)} f_{l,t}^* - \sum_{l \in to(i)} f_{l,t}^*$$

is a supergradient of L in π; i.e. g $\in \partial L(\pi)$

An example: the subgradient algorithm

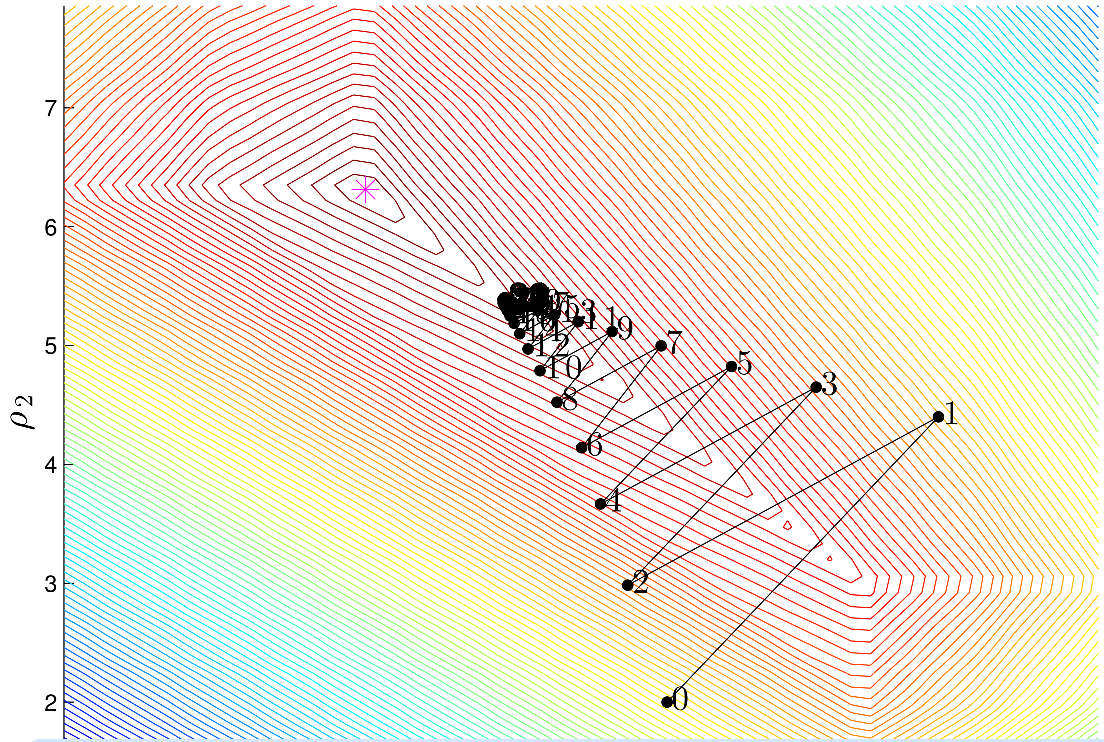
Toy example [6]	
Demand: [30, 40] *MW*	*Two-periods, single node*

G	C_g^P	C_g^{SU}	$P_g^{\min}$	$P_g^{\max}$
SMOKESTACK01	*3*	*53*	*0*	*16*
SMOKESTACK02	*3*	*53*	*0*	*16*
SMOKESTACK03	*3*	*53*	*0*	*16*
HIGH_TECH01	*2*	*30*	*0*	*7*
HIGH_TECH02	*2*	*30*	*0*	*7*
MED_TECH01	*7*	*0*	*2*	*6*

GENERIC ALGORITHMIC SCHEME

1. Given a price π_k, $L(\pi_k)$ and $\partial L(\pi_k)$ are evaluated
2. Given this information, a new price π_{k+1} is generated
3. If stopping criterion, stop. Otherwise, go to 1

Lagrangian value function and Subgradient path



- **Supgradient** algorithm is **memoryless**
- Typical oscillation behavior

→ In moderate dimension, such as for our CHP problem, there are more optimistic algorithmic schemes

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The Kelley's algorithm [7]

- Basis for the Level Method
- Based on the idea of iteratively constructing a **model**: the piecewise linear function $L(\pi)$ is **upper-approximated** at each iterate by a model function $\hat{L}(\pi, k)$ consisting of **supporting hyperplanes**

Model function

$$\hat{L}(\pi, k) = \min_{j=0..k} \left[\langle g_j, \pi - \pi_j \rangle + L(\pi_j)\right] \tag{5}$$

Master program

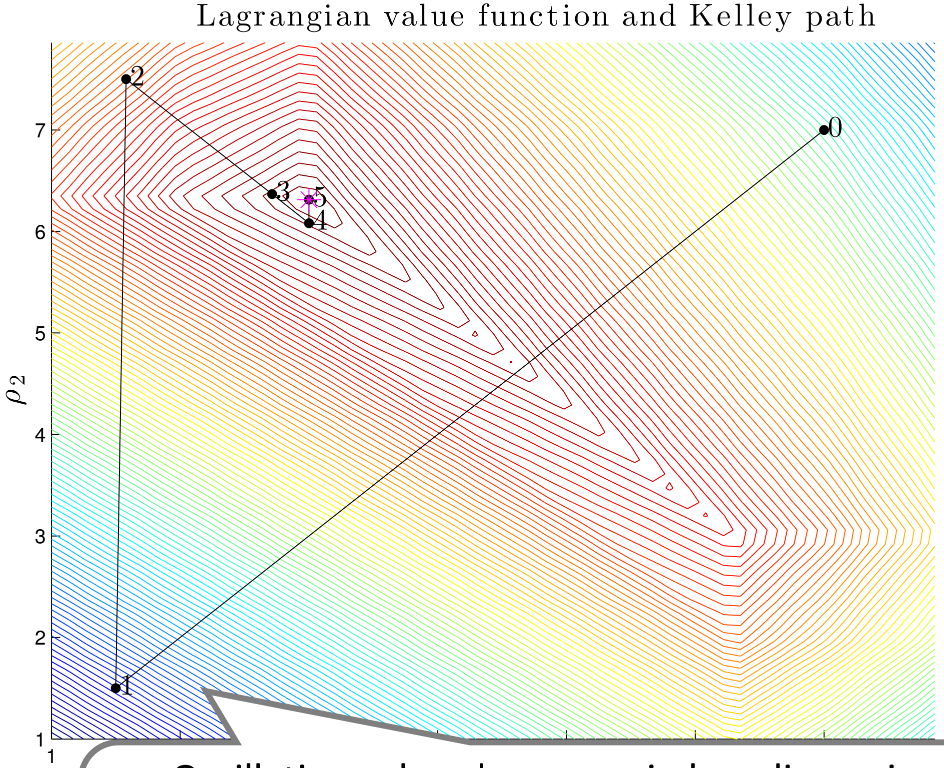
$$\begin{aligned} \max_{\pi \in Q, \theta} \quad & \theta \\ s.t. \quad & \theta \leq \langle g_j, \pi \rangle + b_j \quad \forall j = 0..k \end{aligned} \tag{6}$$

Update rule

$$\pi_{k+1} = \arg\max_{\pi} \hat{L}(\pi, k). \tag{7}$$

Stopping criterion

$$\frac{UB_k - LB_k}{|UB_k|} \leq \epsilon \tag{8}$$



- Oscillations already appear in low dimension
- **Unstable** in high dimension: adding a new supporting hyperplane can move the optimum far from the previous point

The Level stabilization [7]

- The underlying idea of the Level Method is to **update prices more smoothly**
- Select the next iterate π_{k+1} so that it is better than π_k (as evaluated by $\hat{L}(\pi, k)$), without being optimal at all costs

Same **Model function** as Kelley

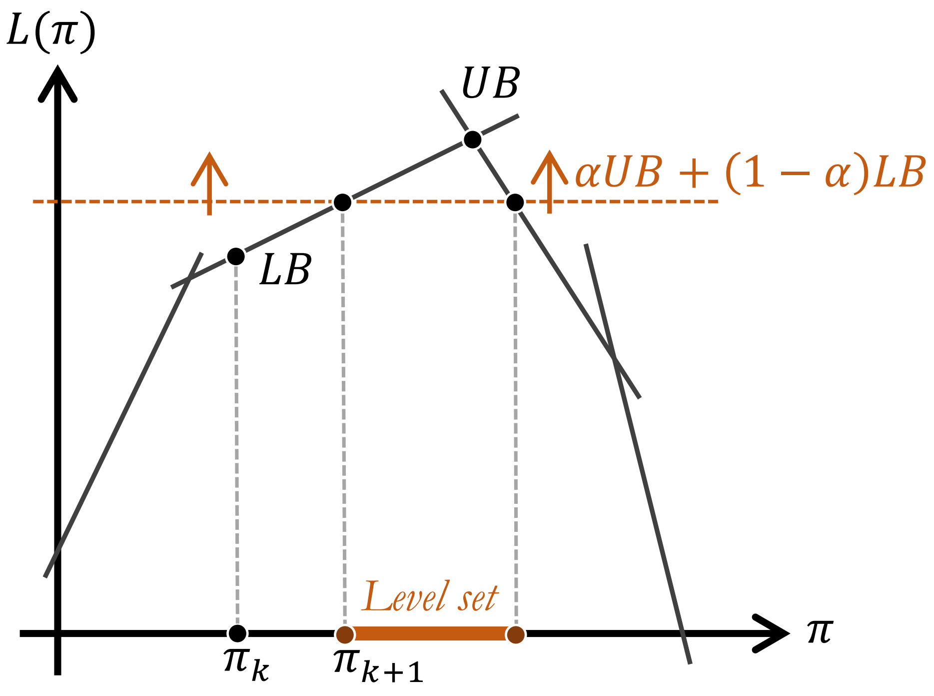
Same **Master program** as Kelley

NEW **Update rule** Projection prog.

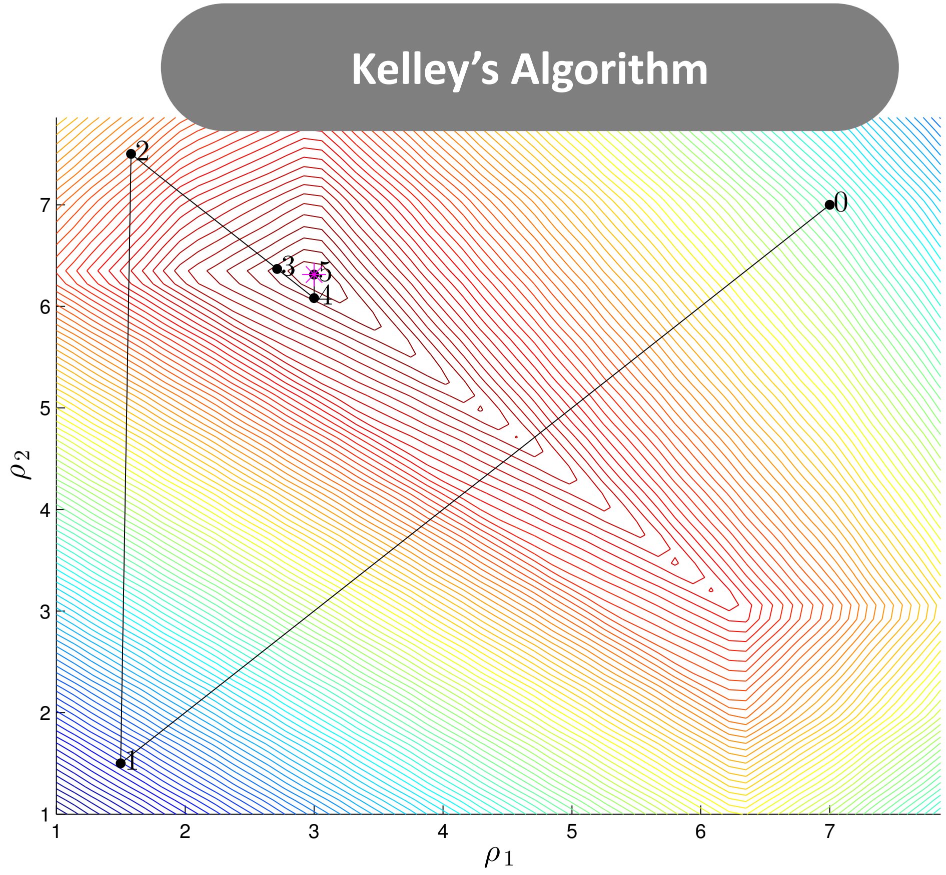
$$\min_{\pi \in Q} \quad ||\pi - \pi_k||_2^2$$
$$s.t. \quad \langle g_j, \pi \rangle + b_j \geq \alpha UB_k + (1-\alpha) LB_k \quad \forall j = 0..k \tag{9}$$

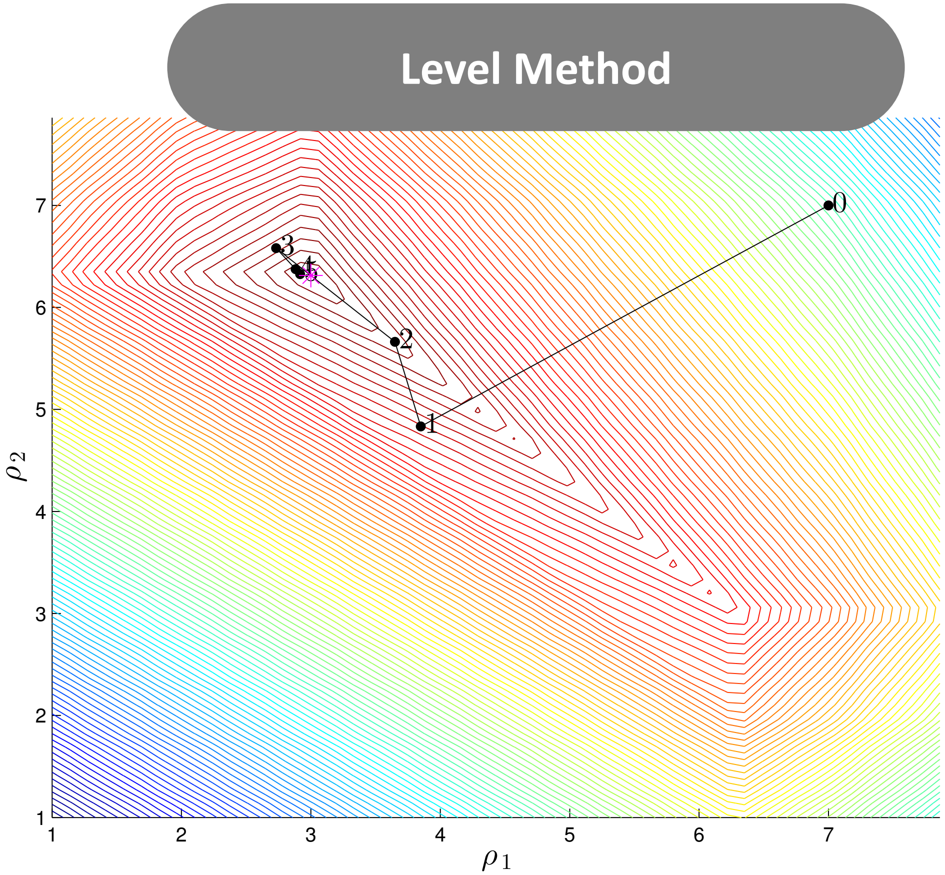
α is the **projection parameter**
- $\alpha = 1$: Kelley's algorithm
- $\alpha = 0$: the iterate does not move

Same **Stopping criterion** as Kelley



The Level Method stabilizes Kelley's path – illustration on 2D example





Two adaptation of the Level Method to the CHP specificities

Two tricks can improve the classical Level formulation to the specificities of the CHP problem

$$L(\pi) = \sum_{i,t} \pi_t^i D_t^i \tag{4a}$$

$$-\sum_{g\in\mathcal{G}} \max_{(c,p,u)_g\in\mathcal{X}_g} \left\{ \sum_t p_{g,t}\pi_t^{i(g)} - c_g \right\} \tag{4b}$$

$$+\min_{f\in\mathcal{F}} \left\{ \sum_{i,t} \pi_t^i \left(\sum_{\substack{l\in\\ from(i)}} f_{l,t} - \sum_{l\in to(i)} f_{l,t} \right) \right\} \tag{4c}$$

1. All generators are separable → write one upper approximation θ_g per generator (one cut per generator)

2.
- Dualize the convex network equations
- Insert it back into the Lagrangian function (i.e. it becomes explicit variable of the master & projection)

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Computing CHP in the European day-ahead auction

Requirements [5]

- Euphemia is afforded **12 minutes** of run time
- The market model includes a network of $\sim$ **40 bidding zones**, and its geographic footprint is expected to be further enlarged
- The market model is expected to move towards **15-minute granularity** by 2022 (a horizon of **96 periods**)

Test cases

- We focus on the sensitivity of the algorithms towards the **dimension of the price space** (the ultimate goal is to compute **prices** optimizing $L(\pi)$)
- Tested on 2 sets of cases
  - **FERC data** (11 instances, no network, >900 generators) [8]
  - **Central Europe data** (6 times series, 2 different networks) [9]
- The Level Method is benchmarked against a **Dantzig-Wolfe algorithm** [10]

TABLE II
DESCRIPTION OF THE SIZE OF THE EU INSTANCES.

Test case	Bidding Zones	Lines	Generators
BE	30	30	74
BE-NL	59	63	145

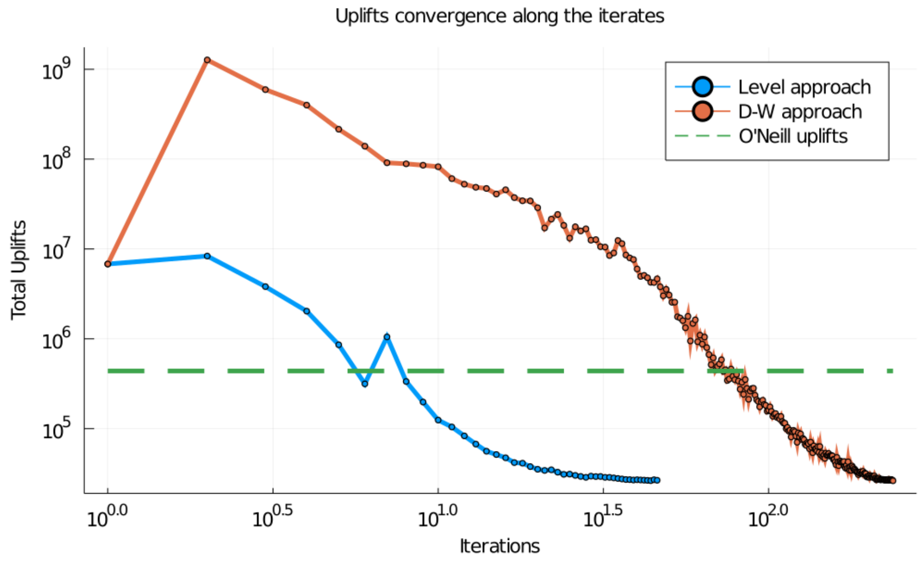
Comparison of the Level Method and Dantzig-Wolfe algorithm

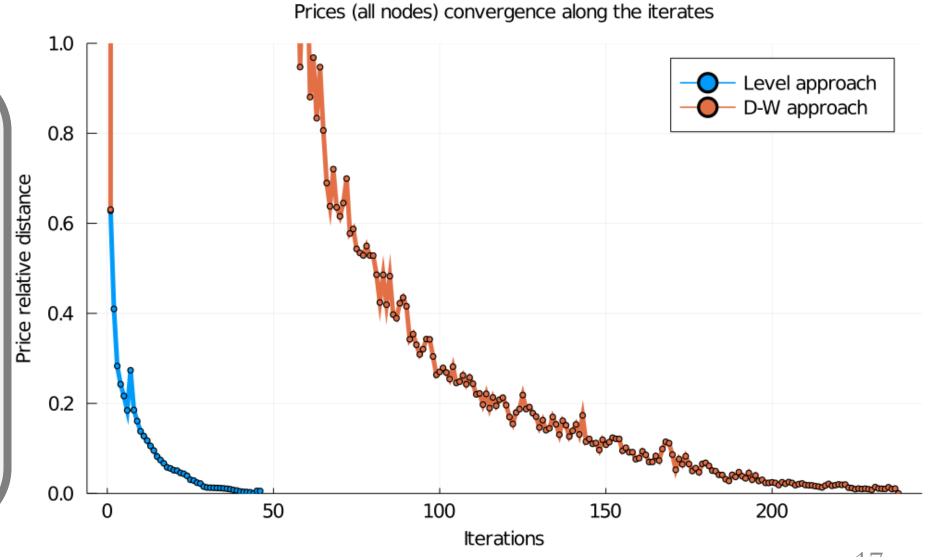
TABLE III
RESULTS OF THE LEVEL METHOD AND THE DANTZIG-WOLFE ALGORITHM ON THE BE TEST CASE FOR DIFFERENT TIME HORIZONS (AVERAGE OVER 6 INSTANCES).

horizon	Dispatch Cost [€]	IP Uplifts [€]	CHP Uplifts [€]	Level iter	Level av. time per iter[a][s]	D-W iter	D-W av. time per iter[a][s]
12	2,767,841	398,003	6,621	22	0.5 (0.05)	18	0.3 (0.02)
24	4,963,246	144,125	10,149	28	0.8 (0.1)	39	0.6 (0.1)
48	11,335,441	288,741	20,746	32	1.9 (0.4)	78	1.9 (0.3)
96	24,104,303	2,587,632	28,987	46	6.4 (1.8)	241	6.5 (2.1)

[a] (·) denotes the average time per iterate for solving the "master programs" (i.e. master plus projection in the case of the Level Method).

- Implementation
  - Julia (JuMP)
  - Run on a personal computer (Intel Core i5, 2.6 GHz, 8 GB of RAM)
  - Gurobi 9.1.1
  - Stopping criterion: 0.01%
- **Iteration count**: < 50 iterates
- **Robustness**: the Level Method reaches quickly good price candidates (valuable since clearing time limited to 12 min)





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Conclusion

- The Level Method exhibits **favorable empirical performance** to solve Convex Hull Pricing problem

- It is capable to **compute Convex Hull Prices on large instances** including network and a horizon of 96 periods (which anticipates future EU DA market evolution)

- Further promising paths of research:
  1. **Expanding tests** on realistic instances of Euphemia
  2. Examining the **effects of non-uniform pricing on enhancing welfare** in the EU day-ahead market
  3. Understanding **distributional effects** of non-uniform pricing as well as **gaming effects**

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Thank you!

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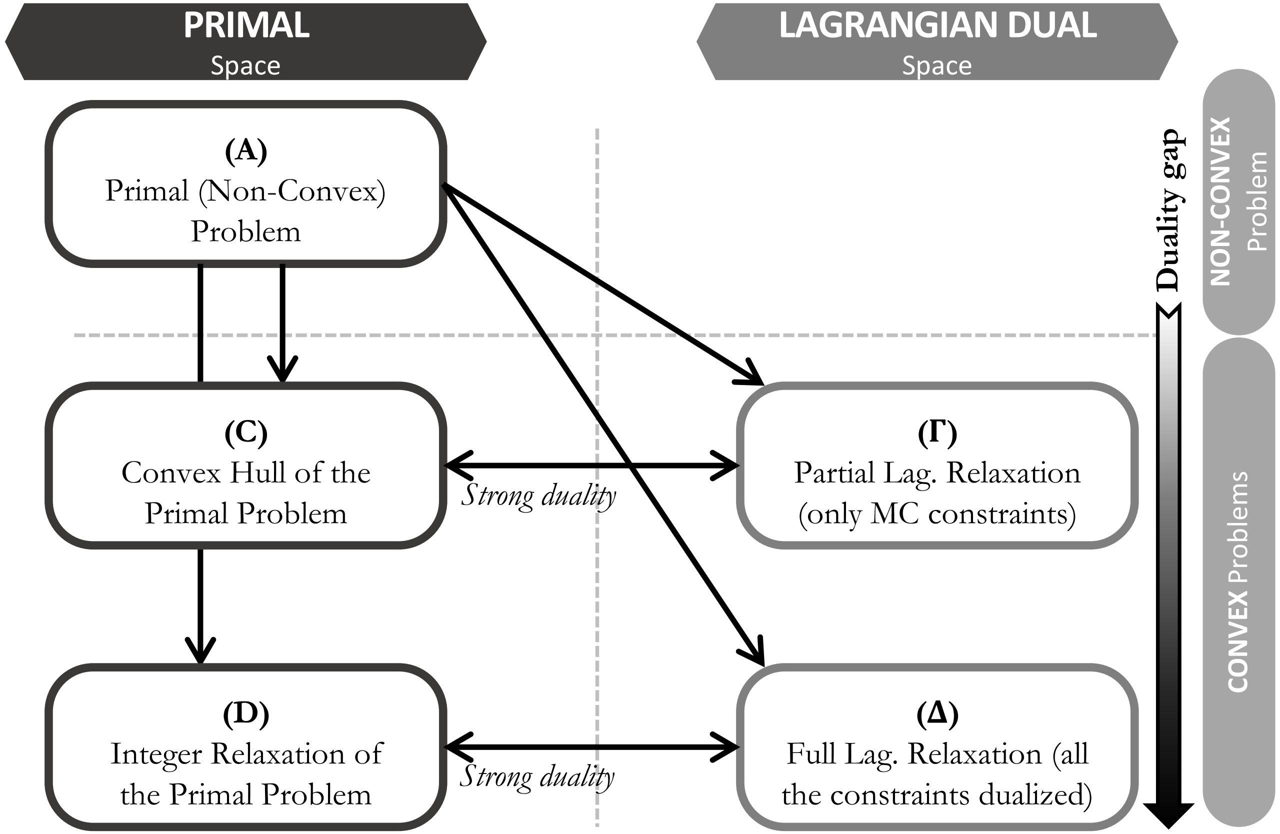
Landscape of CHP related problems



Adaptation of the Level Method to the CHP specificities (1) – network

We can exploit the convexity of the network model and leverage it in the formulation of the master and projection programs

$$L(\pi) = \sum_{i,t} \pi_t^i D_t^i \tag{4a}$$

$$- \sum_{g\in\mathcal{G}} \max_{(c,p,u)_g \in \mathcal{X}_g} \left\{ \sum_t p_{g,t}\pi_t^{i(g)} - c_g \right\} \tag{4b}$$

$$+ \min_{f\in\mathcal{F}} \left\{ \sum_{i,t} \pi_t^i \left(\sum_{l\in from(i)} f_{l,t} - \sum_{l\in to(i)} f_{l,t} \right) \right\} \tag{4c}$$

Network term

1 Write the network equation (let's consider here the DC voltage angle formulation

$$\min_{f,\psi} \sum_{i,t} \pi_t^i \left(\sum_{l\in from(i)} f_{l,t} - \sum_{l\in to(i)} f_{l,t} \right) \tag{10a}$$

$$(\mu_{l,t}) \quad f_{l,t} \leq \overline{F}_l \qquad \forall l,t \tag{10b}$$

$$(\nu_{l,t}) \quad f_{l,t} \geq \underline{F}_l \qquad \forall l,t \tag{10c}$$

$$(\lambda_{l,t}) \quad f_{l,t} = B_l(\psi_{or(l),t} - \psi_{dest(l),t}) \qquad \forall l,t \tag{10d}$$

2 Derive the **dual** of the convex problem

$$\max_{\mu\geq 0,\nu\geq 0,\lambda} \sum_{l,t} \nu_{l,t}\underline{F}_l - \mu_{l,t}\overline{F}_l \tag{11a}$$

$$\pi_t^{or(l)} - \pi_t^{dest(l)} + \mu_{l,t} - \nu_{l,t} + \lambda_{l,t} = 0 \qquad \forall l,t \tag{11b}$$

$$\sum_{l\in to(i)} \lambda_{l,t}B_l - \sum_{l\in from(i)} \lambda_{l,t}B_l = 0 \qquad \forall i,t \tag{11c}$$

3 Substitute it to (4c) in the master and projection programs

Adaptation of the Level Method to the CHP specificities (2) – multi-cuts

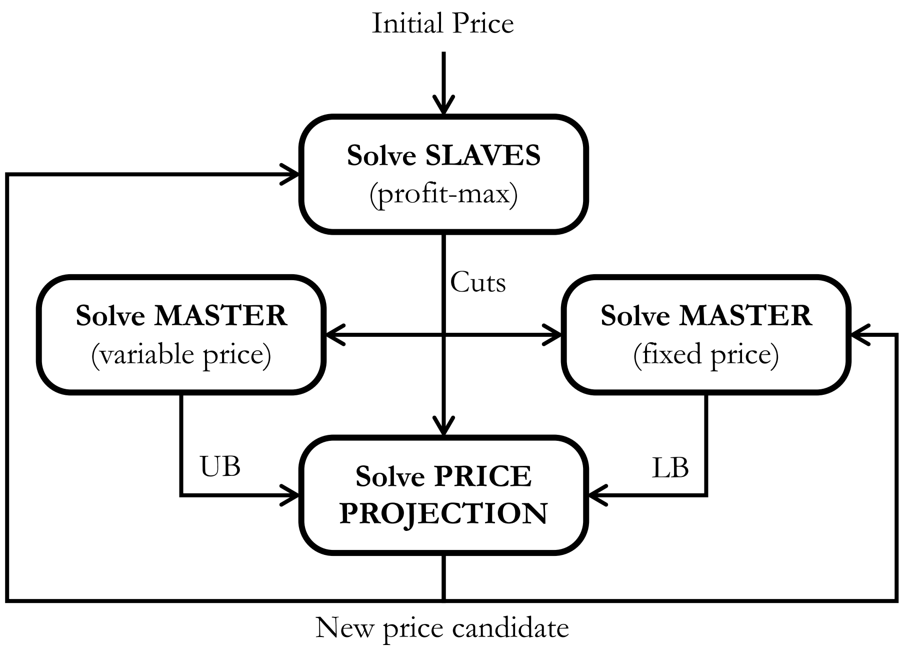
- The classical Kelley/Level Method adds one cut per iterate
- Instead, we can add **one cut per agent**
- Our experiments reveal that this adaptation can deliver substantial computational benefits.

$$\max_{\substack{\mu\geq 0,\nu\geq 0,\\ \lambda,\pi\in Q,\theta}} \sum_{i,t} \pi_t^i D_t^i + \sum_{l,t}\left(\nu_{l,t}\underline{F}_l - \mu_{l,t}\overline{F}_l\right) - \sum_{g\in\mathcal{G}} \theta_g \tag{12a}$$

$$\theta_g \geq \langle p_{g,\cdot}^j, \pi^{i(g)} \rangle - c_g^j \quad \forall g, j = 0..k \tag{12b}$$

$$\pi_t^{or(l)} - \pi_t^{dest(l)} + \mu_{l,t} - \nu_{l,t} + \lambda_{l,t} = 0 \quad \forall l, t \tag{12c}$$

$$\sum_{l\in to(i)} \lambda_{l,t} B_l - \sum_{l\in from(i)} \lambda_{l,t} B_l = 0 \quad \forall i, t \tag{12d}$$



(1) Network treatment

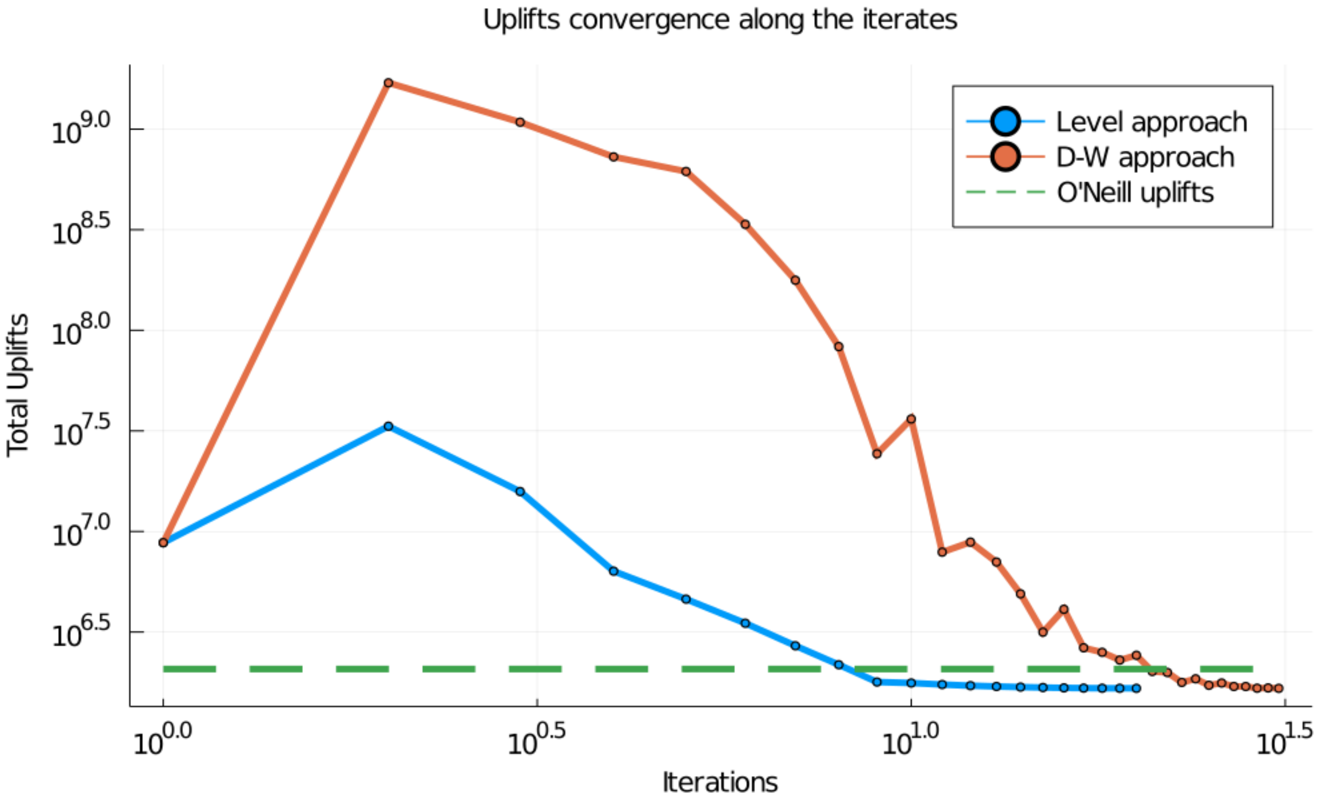
(2) Multi-cut scheme

FERC test case results

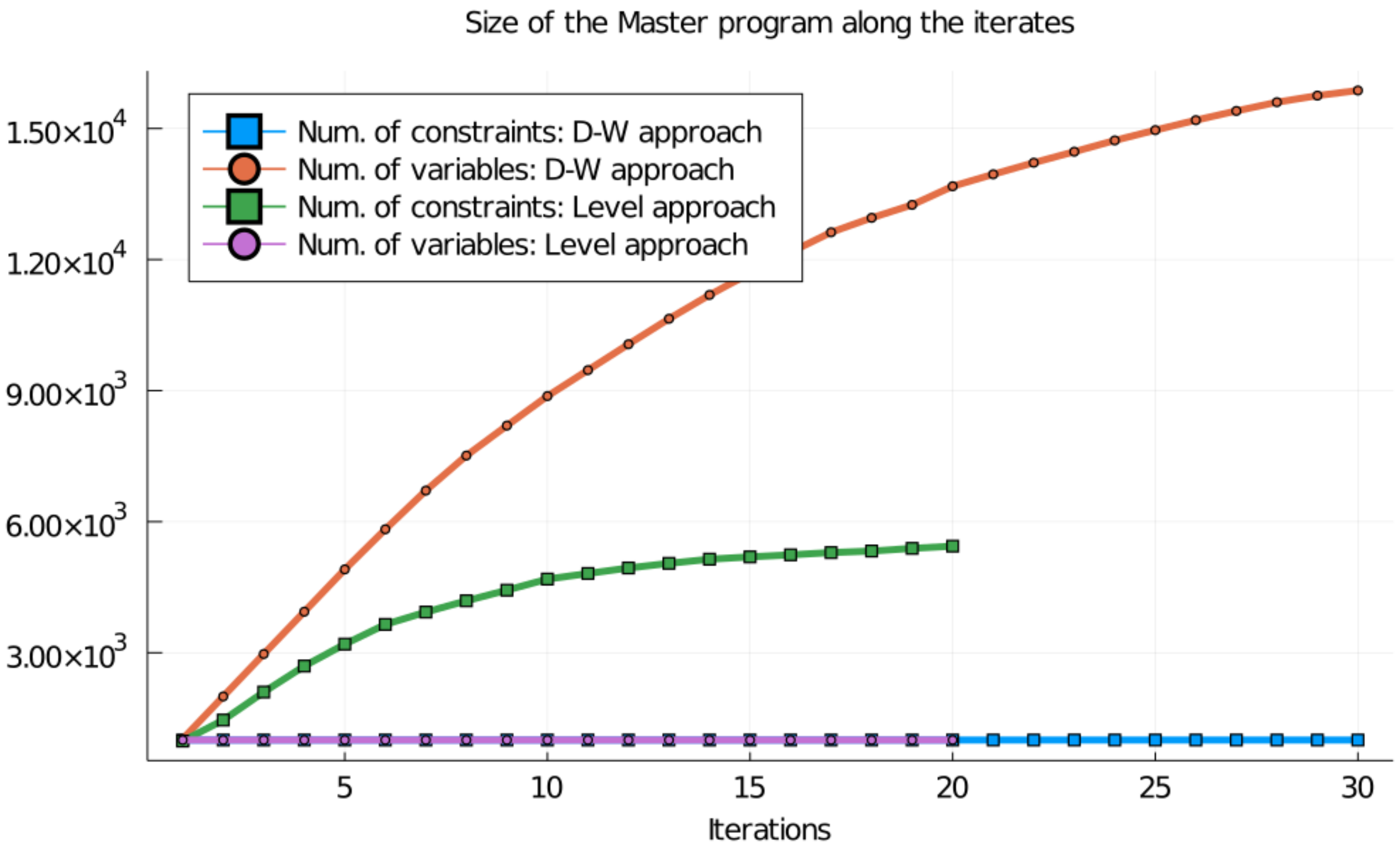
TABLE I
RESULTS OF THE LEVEL METHOD AND THE DANTZIG-WOLFE ALGORITHM ON FERC DATASETS (AVERAGE OVER 11 INSTANCES).

Dispatch Cost [$]	IP Uplifts [$]	CHP Uplifts [$]	Level iter	Level av. time per iter[a][s]	D-W iter	D-W av. time per iter[a][s]
30,558,573	1,247,219	947,551	19	8.3 (0.3)	29	9.0 (0.3)

[a] $(\cdot)$ denotes the average time per iterate for solving the "master programs" (i.e. master plus projection in the case of the Level Method).



FERC test case results – comparison of the master sizes



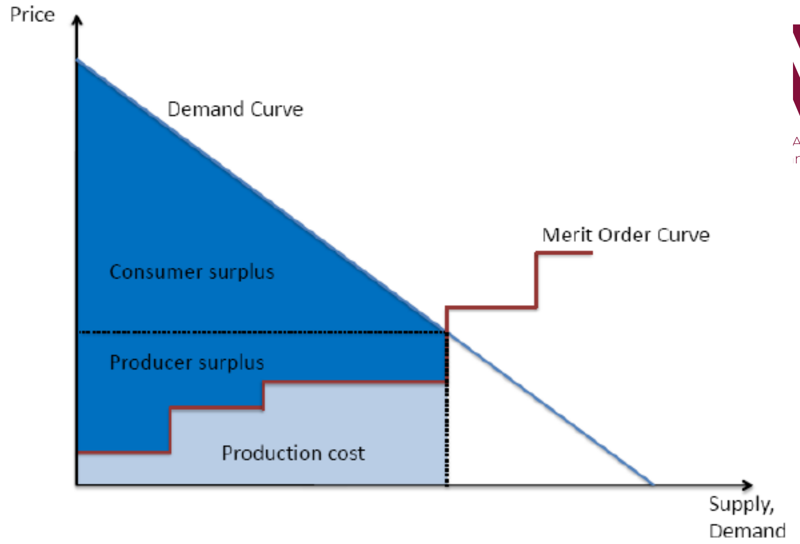
Comparison of the Level Method and Dantzig-Wolfe algorithm – network

TABLE IV
RESULTS OF LEVEL METHOD AND THE DANTZIG-WOLFE ALGORITHM FOR DIFFERENT NETWORK SIZES (AVERAGE OVER 6 INSTANCES).

horizon	Test case	IP Uplifts [€]	CHP Uplifts [€]	Level iter	Level av. time per iter [s]	D-W iter	D-W av. time per iter [s]
24	**BE**	144,125	10,149	28	0.8 (0.1)	39	0.6 (0.1)
24	**BE-NL**	105,045	12,418	21	1.5 (0.3)	32	1.3 (0.2)
96	**BE**	2,587,632	28,987	46	6.4 (1.8)	241	6.5 (2.1)
96	**BE-NL**	3,024,015	34,615	42	12.3 (4.8)	156	12.7 (3.7)

[a] $(\cdot)$ denotes the average time per iterate for solving the “master programs” (i.e. master plus projection in the case of the Level Method).

EU MARKET DESIGN IN A NUTSHELL

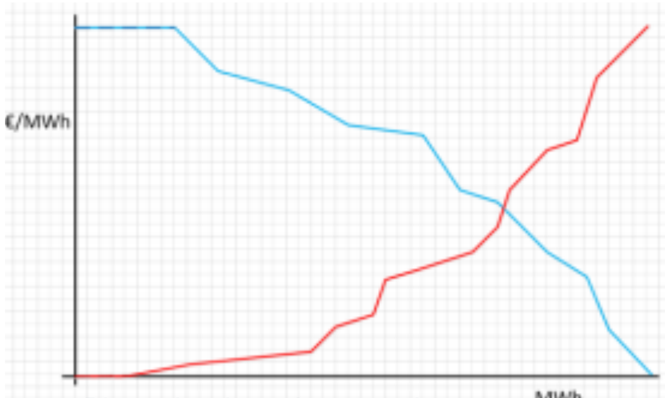


- Objective: Maximise the **welfare** defined as the sum of:
  - — Surplus of the buyers
  - — Surplus of the sellers
  - — Congestion rent
- Constraints #1: Considering the **network constraints**
  - — **Zonal pricing**: one country ~= 1 bidding zone (>< nodal pricing in the US)
  - — The network (cross-border flows) is currently constrained by:
    - Flow-based constraints (PTDF) — CWE region
    - Available transfer capacities (ATC) — the others
- Constraints #2: Considering **market orders** (Buy and sell orders)
  - — **Portfolio bidding**: market participants submit generic financial orders (which can hide a mixture of multiple power plants) → the complex power plant characteristics are represented through generic financial orders
  - — >< US model which is **unit bidding**: the market participant submit the technical characteristics of their power plants (min up/down time, startup cost, etc.)

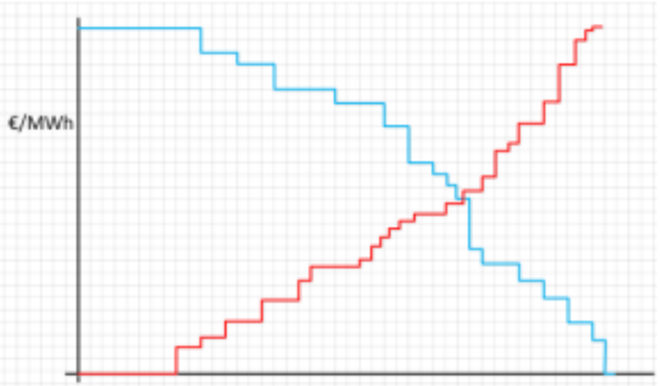
- Multiple types of orders:
  - **Hourly orders**
  - **Block orders**
  - Complex orders
  - PUN, merit order, etc.
- In this presentation, we will focus on these aspects
- And particularly the **pricing strategy** considering these orders

1. HOURLY ORDERS

- The simplest order the participants can auction is the hourly order.



Piecewise linear hourly order



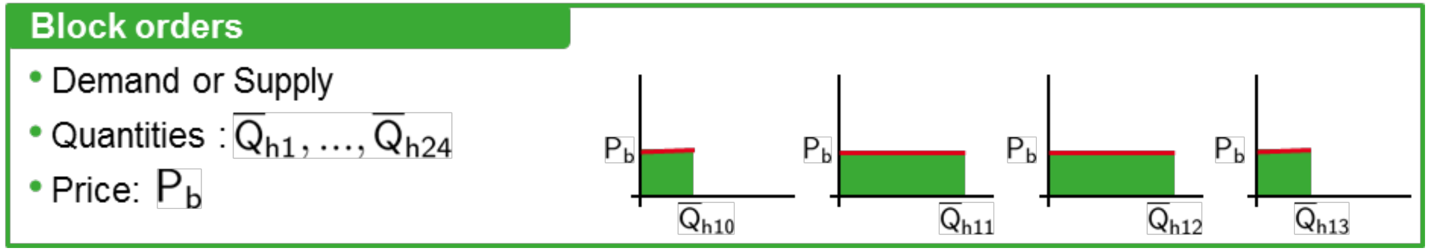
Stepwise hourly order

→ Convex offers (left = quadratic, right = linear)

- The hourly order can be partially accepted (between zero and the order maximum quantity)
- The pricing rule of hourly order is standard and maximizes the profit of every participant
  - Hourly orders in-the-money **must be** accepted.
  - Hourly orders out-the-money **must be** rejected
  - Hourly orders at-the-money **can be** accepted

2. BLOCK ORDERS

- A block order is defined over several periods, its volume can vary during the period (profile) and must be fully accepted or rejected (All-or-Nothing condition).



→ non-convex (MIP) order!

- The pricing rule for block order is the following
  - Block orders in-the-money **can be** accepted
  - Block orders out-the-money **must be** rejected
- Block order in-the-money can be rejected (Paradoxically rejected order – loss of potential gain).
- In some exchanges (eg. NORDPOOL), you can also specify a minimum acceptance ratio, allowing the block to be curtailed down to the set Minimum Acceptance Ratio (➔ more flexibility).