

# Price Formation with Non-Convexities

Theory and Applications for the Electricity Market

Nicolas Stevens

PhD Public Defense, December 2024

**Supervisors**

Anthony Papavasiliou (NTUA, Greece)  
Bert Willems (UCLouvain, Belgium)

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1

*The Electricity sector*

2

*Electricity market & price*

3

*Non-convexities & market price*

4

*Thesis contributions & conclusions*

1

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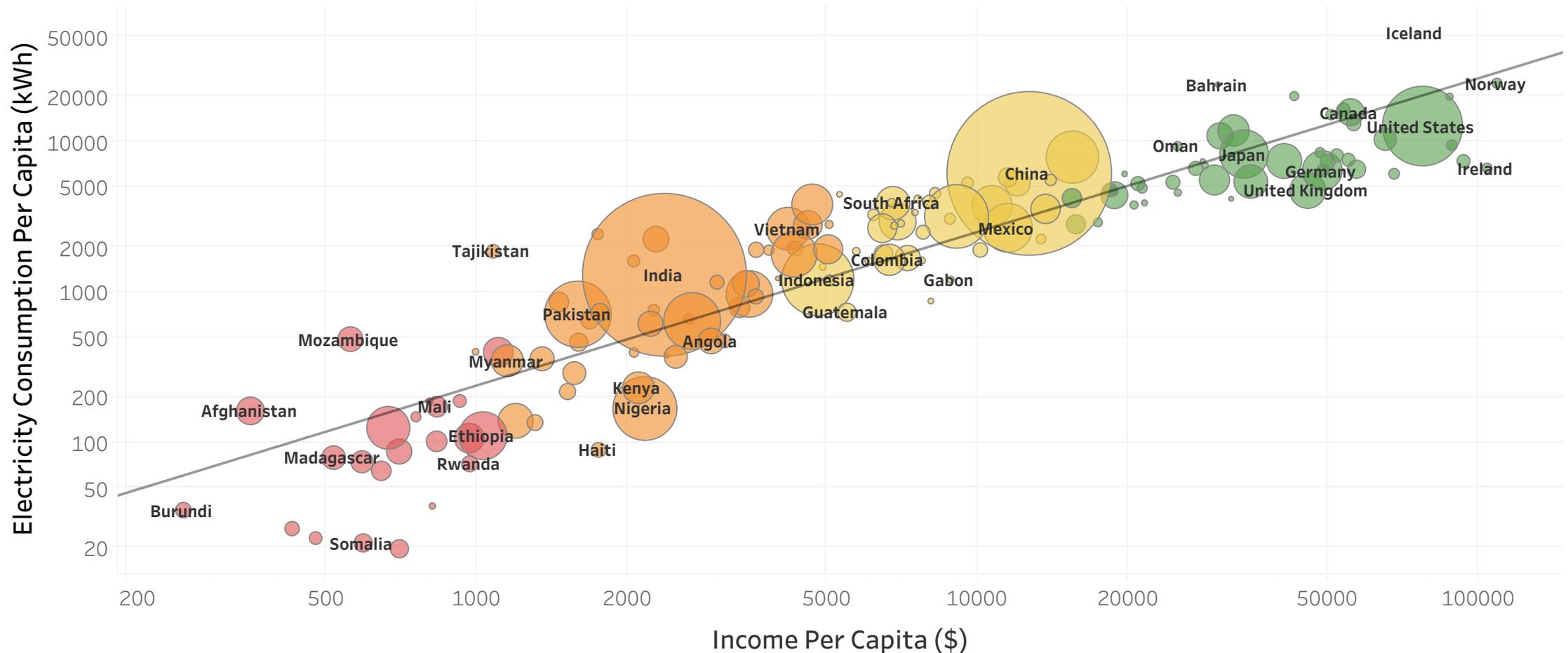
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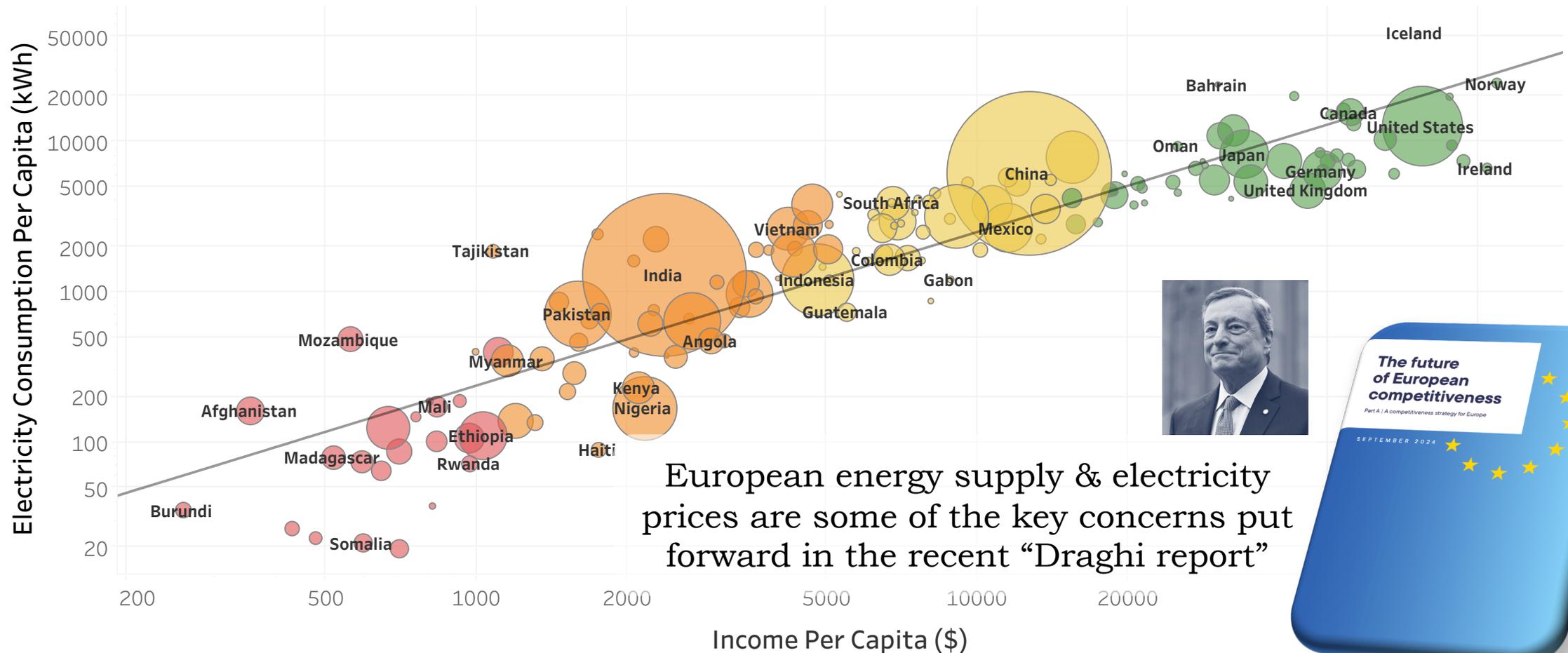
# Electricity is an important sector of our economies

Consumption vs GDP (2022)

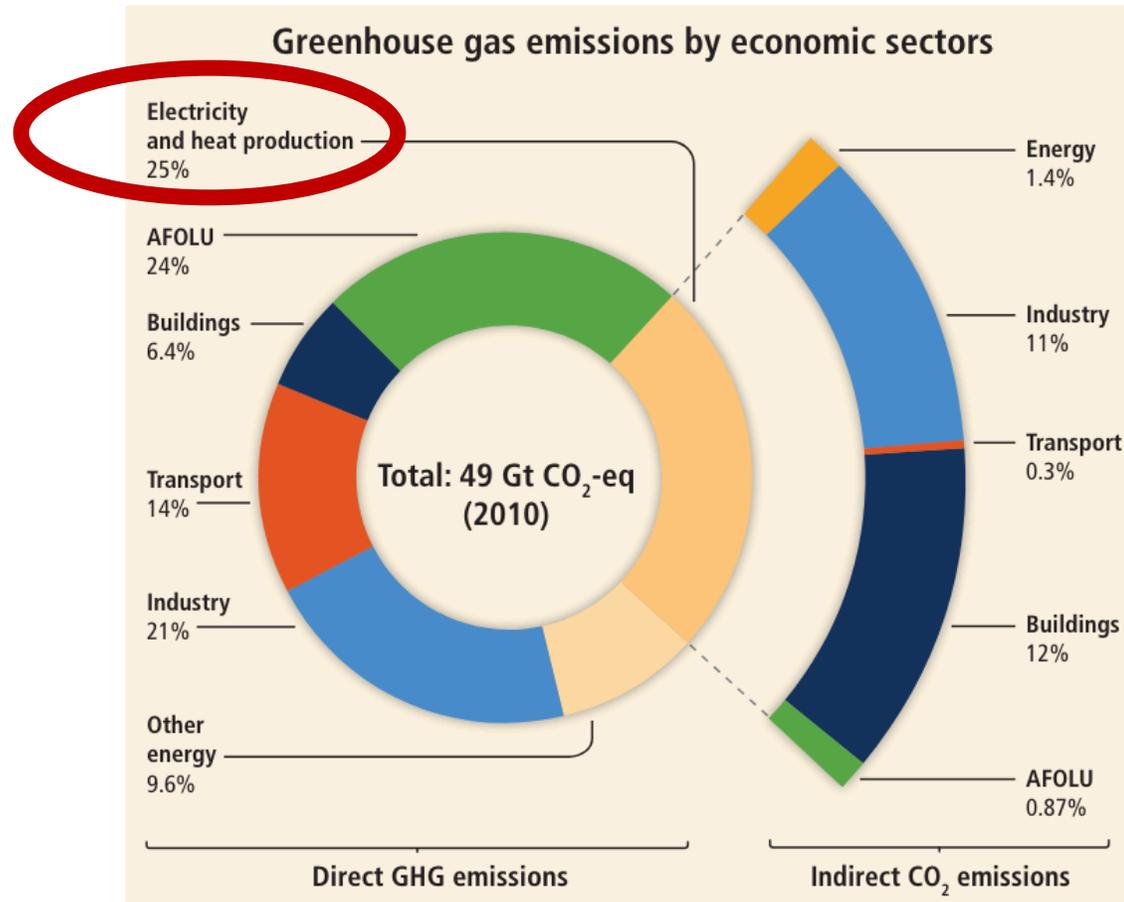


# Electricity is an important sector of our economies

Consumption vs GDP (2022)



# Electricity production stands for ~25% of global CO<sub>2</sub> emissions



**Figure 1.7** | Total anthropogenic greenhouse gas (GHG) emissions (gigatonne of CO<sub>2</sub>-equivalent per year, GtCO<sub>2</sub>-eq/yr) from economic sectors in 2010. The circle shows the shares of direct GHG emissions (in % of total anthropogenic GHG emissions) from five economic sectors in 2010. The pull-out shows how shares of indirect CO<sub>2</sub> emissions (in % of total anthropogenic GHG emissions) from electricity and heat production are attributed to sectors of final energy use. 'Other energy' refers to all GHG emission sources in the energy sector as defined in WGIII Annex II, other than electricity and heat production {WGIII Annex II.9.1}. The emission data on agriculture, forestry and other land use (AFOLU) includes land-based CO<sub>2</sub> emissions from forest fires, peat fires and peat decay that approximate to net CO<sub>2</sub> flux from the sub-sectors of forestry and other land use (FOLU) as described in Chapter 11 of the WGIII report. Emissions are converted into CO<sub>2</sub>-equivalents based on 100-year Global Warming Potential (GWP<sub>100</sub>), taken from the IPCC Second Assessment Report (SAR). Sector definitions are provided in WGIII Annex II.9. {WGIII Figure SPM.2}

[Source: IPCC 5<sup>th</sup> report (2014)]

\* AFOLU stands for "Agriculture, Forestry and Other Land Use"

# Renewable generation capacity in Belgium

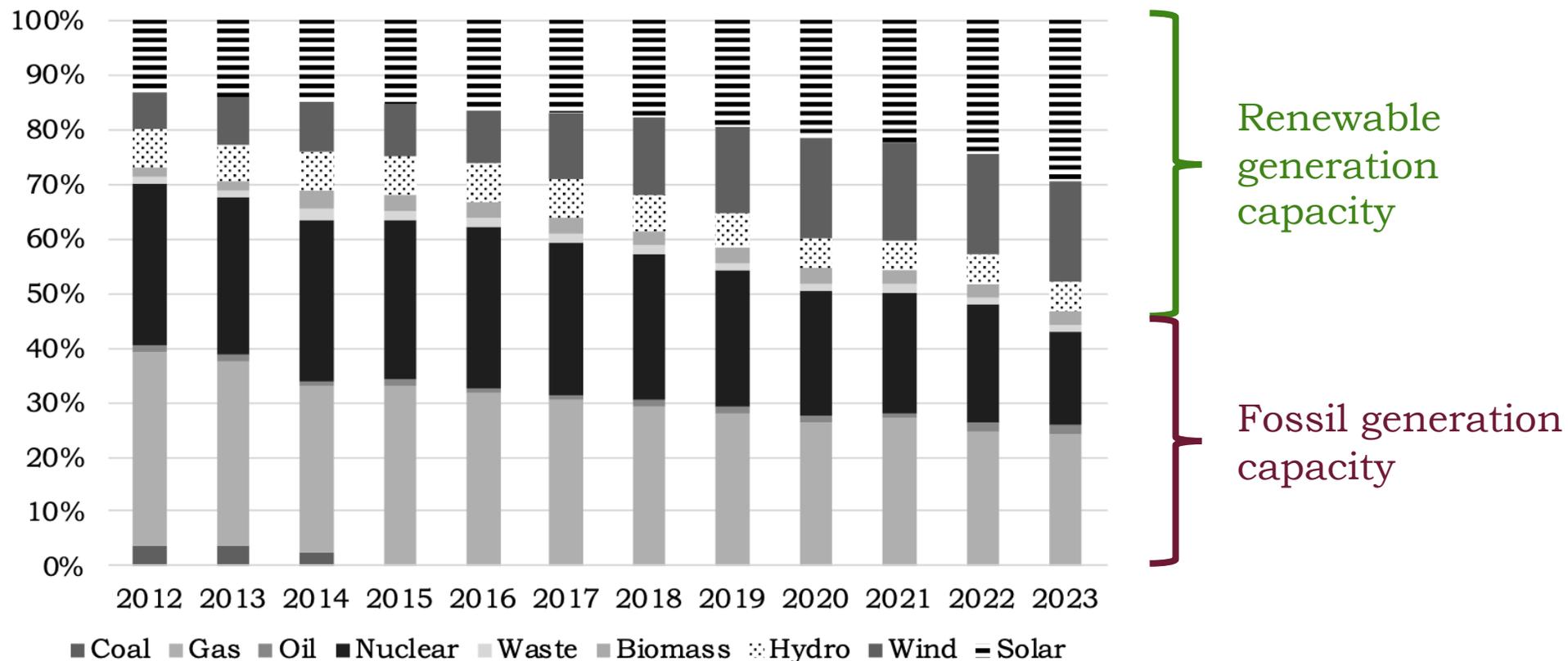
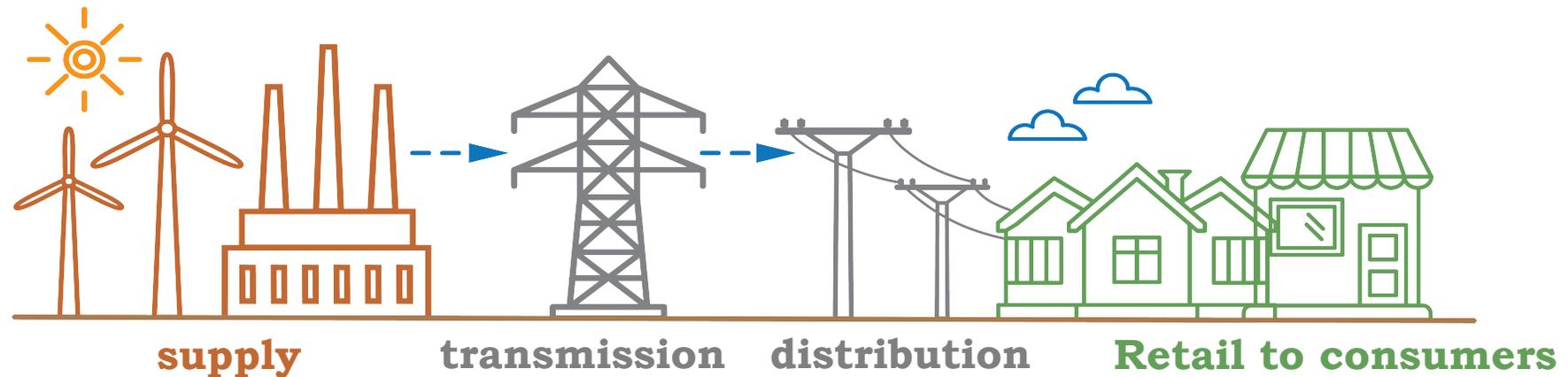


FIGURE 6.1: Electricity generation capacity in Belgium. [Data source: IRENA for wind and PV, Elia for the remaining technologies]

# Electricity fundamentals

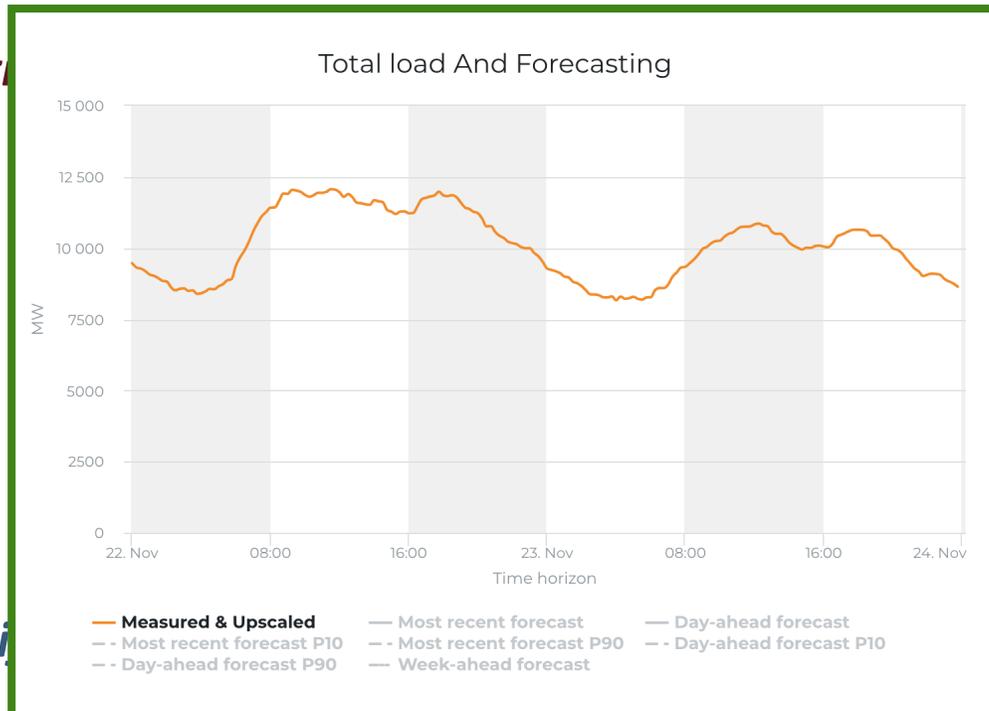
- *Electricity supply chain:*



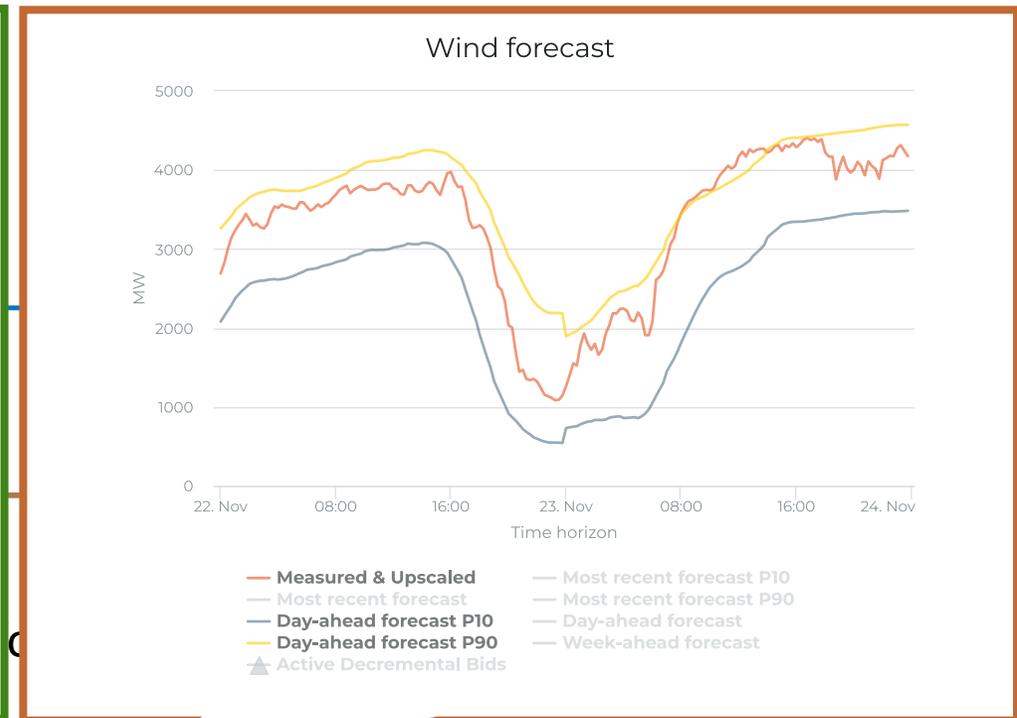
- *Specificities of electricity as a commodity* → need for coordination
  - Exchanged through an **electrical grid** : physical network constraints
  - **Demand** must be met just-in-time by **production**
  - **Difficult / expensive to store**
  - **Low elasticity** of demand (few substitutes & lack of metering)

# Electricity fundamentals

- **Electrification**



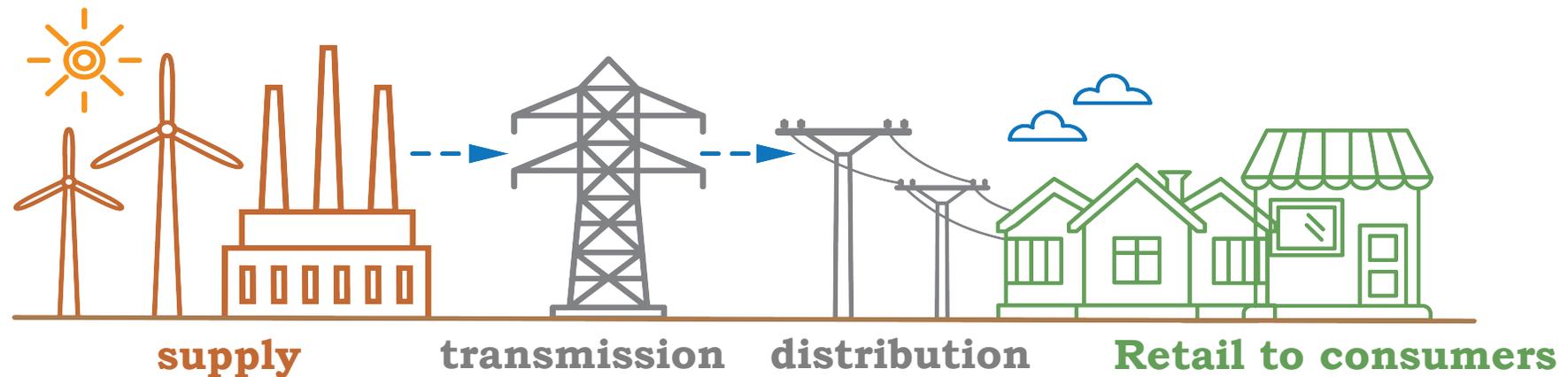
- **Specific**



- Exchange through an **electrical grid** : physical network of units
- **Demand** must be met just-in-time by **production**
- **Difficult / expensive to store**
- **Low elasticity** of demand (few substitutes & lack of metering)

# The liberalization of power systems

- *Electricity supply chain:*

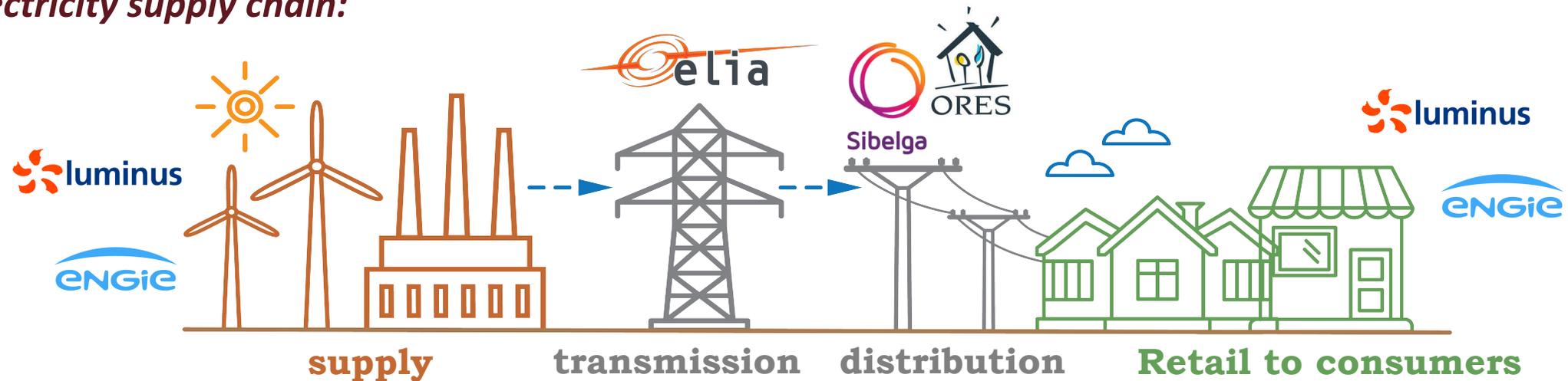


- *Deregulation of power systems:*

- From a **coordination by a single firm** (vertically integrated monopoly) to a **coordination through market transactions** (Coase, 1937)

# The liberalization of power systems

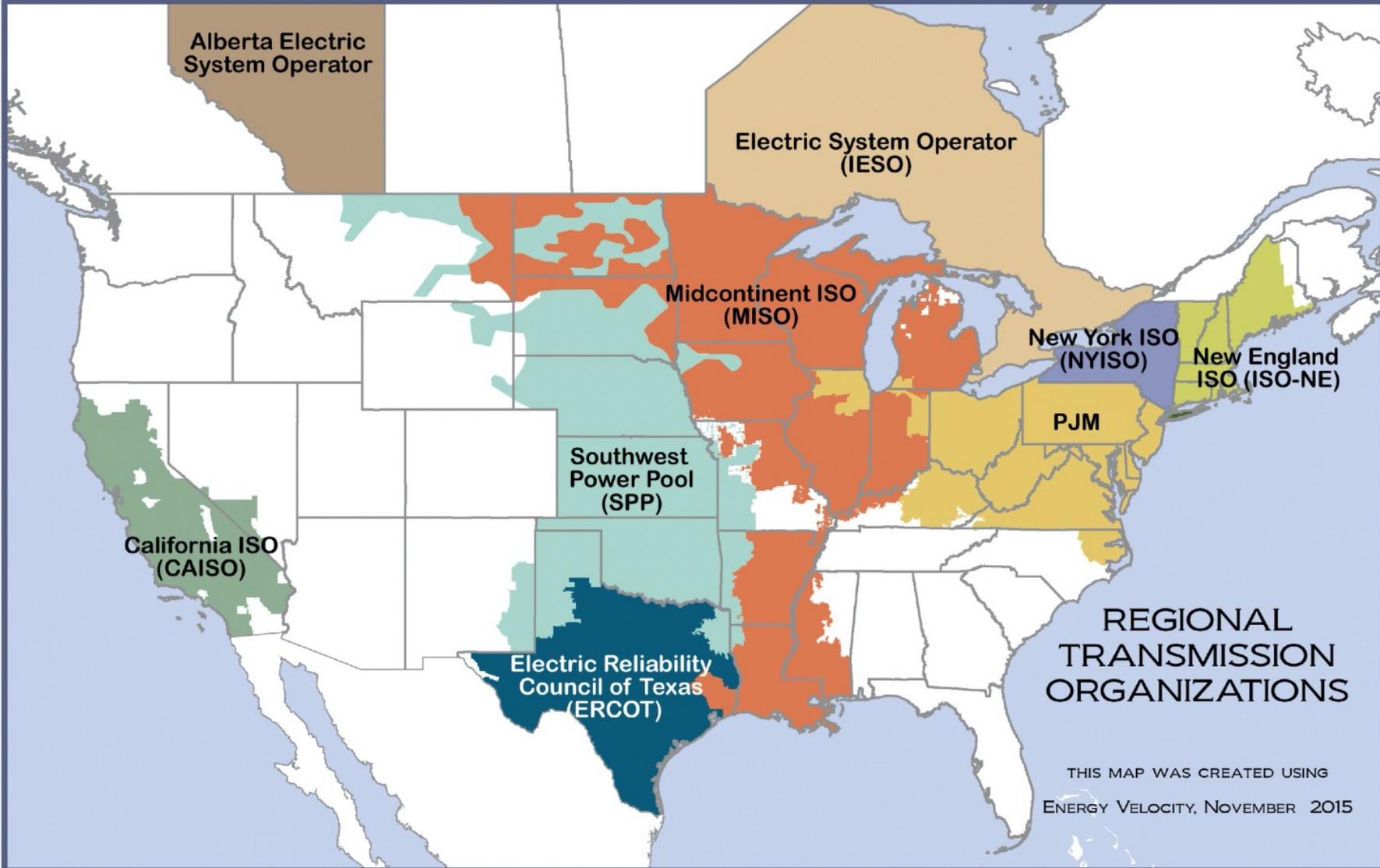
- *Electricity supply chain:*



- *Deregulation of power systems:*

- From a **coordination by a single firm** (vertically integrated monopoly) to a **coordination through market transactions** (Coase, 1937)
- A “vertical” separation of the competitive segments (**generation** and **retail**) from the natural monopoly segments (**transmission and distribution**)
- “the move to liberalizing the electricity sector [...] was effectively a bet that the costs of any residual imperfections in competitive wholesale markets are smaller than the costs of imperfections associated with the behaviour of vertically integrated regulated monopolies.” (Joskow, 2019)

# Electricity markets in the US



1992 : Energy Policy Act

creation of ISOs that operate the market

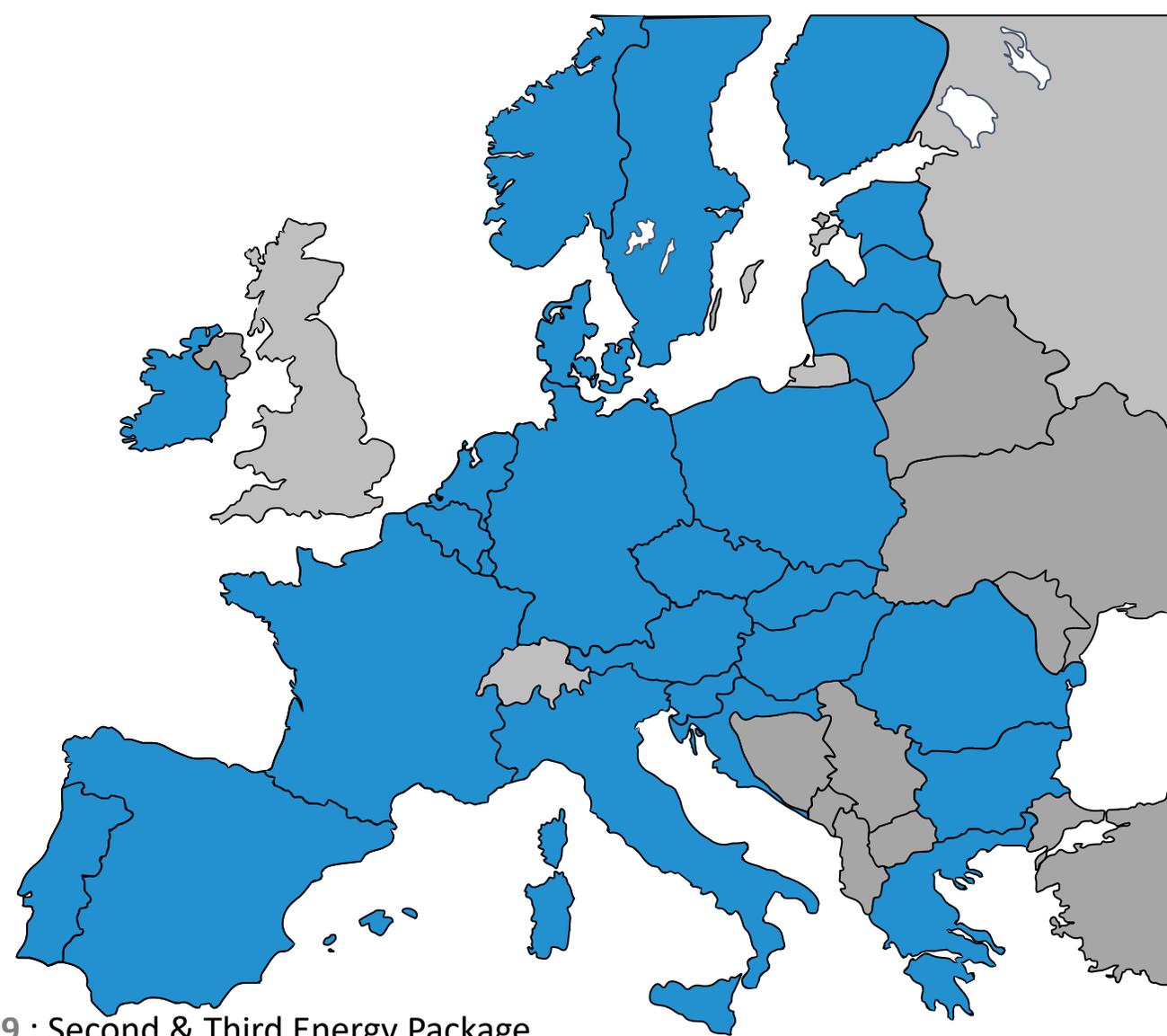
1990

2000

2010

2020

# Electricity markets in Europe



1993 : European Single Market

1996 : First Energy Package (kickoff the liberalization of the power sector)

2003 – 2009 : Second & Third Energy Package (unbundling of competitive and regulated segments )

2006: Trilateral Market Coupling

2014: Single Day-Ahead Coupling (SDAC), Price Coupling of Regions

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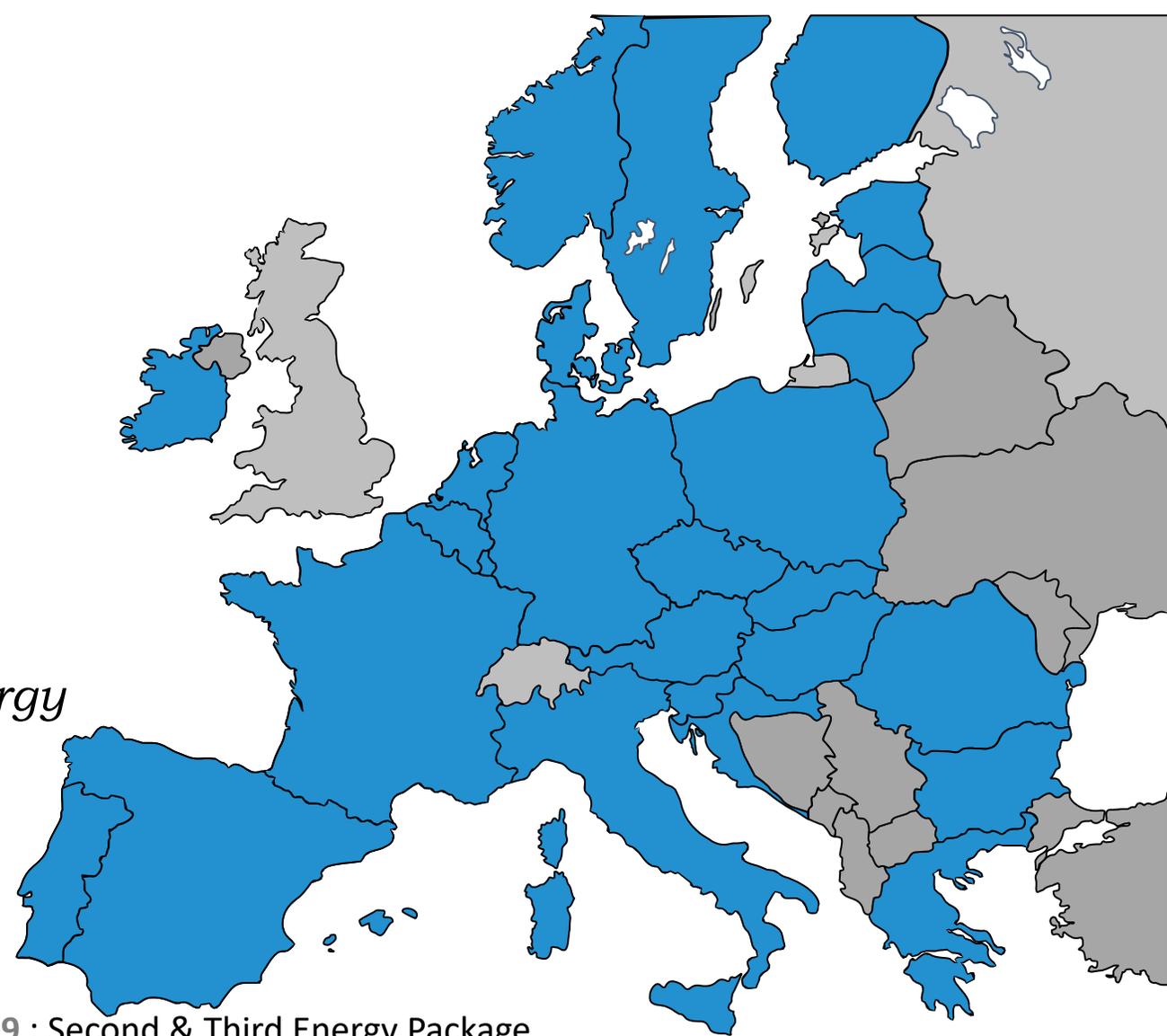
2010

2020

# Electricity markets in Europe

*Liberalization implies that the price of electricity is the outcome of a market...*

*...and not a decision of the Minister of Energy*



1993 : European Single Market

1996 : First Energy Package (kickoff the liberalization of the power sector)

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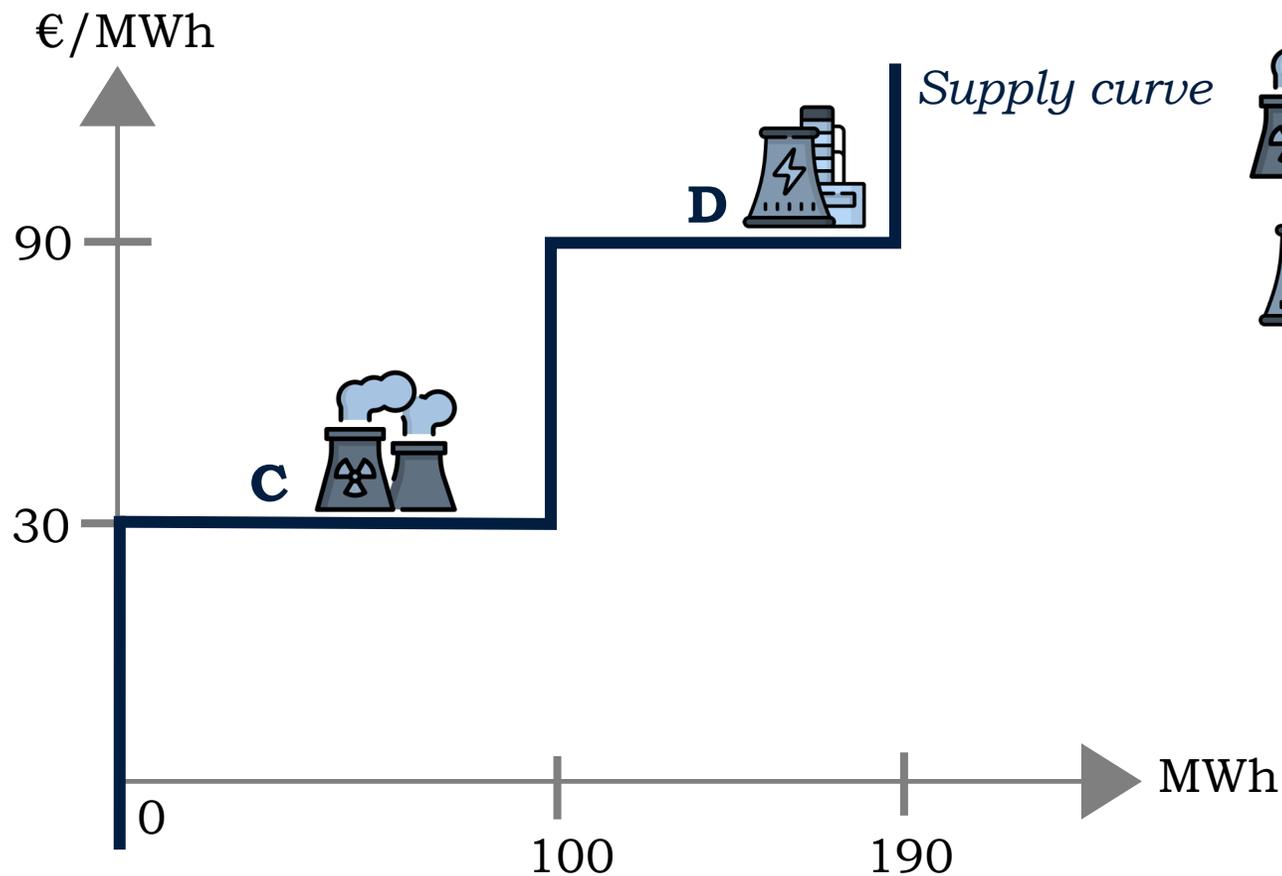
*Non-convexities & market price*

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*Thesis contributions & conclusions*

# Market price fundamentals

## Supply curve

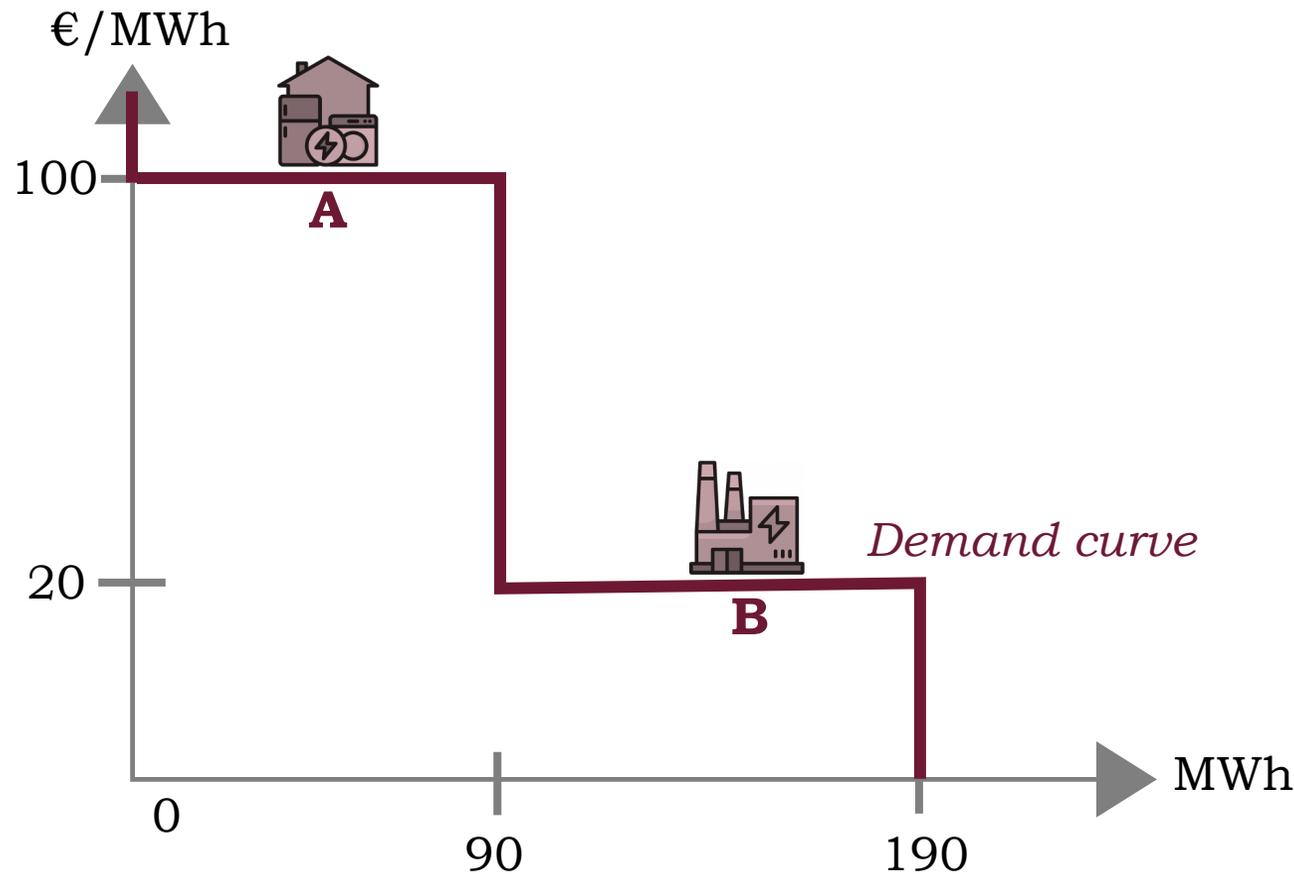


*Supply*

Producer	Capacity [MW]	Cost [€/MWh]
C	100	30
D	90	90

# Market price fundamentals

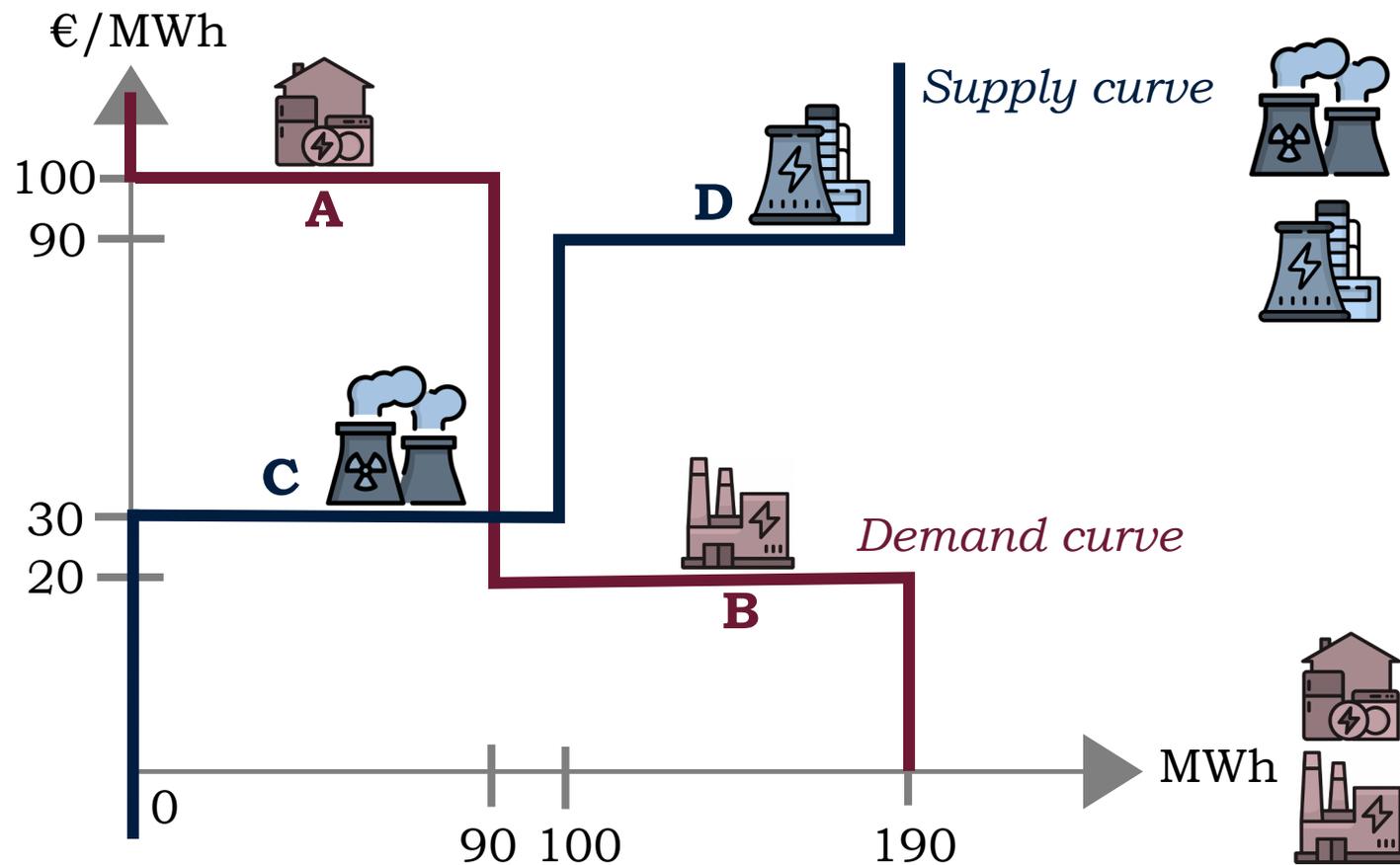
## Demand curve



<i>Demand</i>		
Consumer	Demand Capacity [MW]	Willingness-to-pay [€/MWh]
A	90	100
B	100	20

# Market price fundamentals

## Market equilibrium



*Supply*

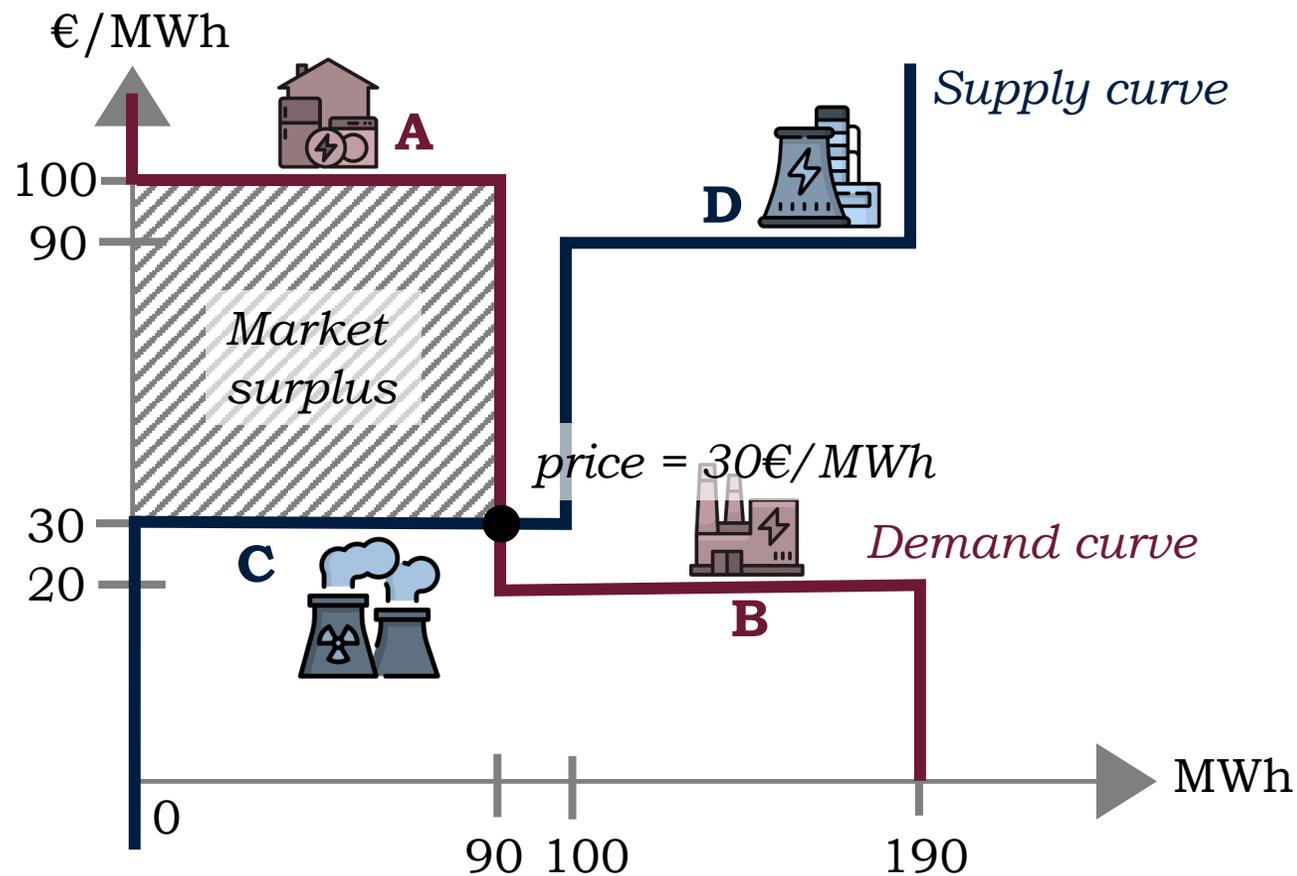
Producer	Capacity [MW]	Cost [€/MWh]
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D	90	90

*Demand*

Consumer	Demand Capacity [MW]	Willingness-to-pay [€/MWh]
A	90	100
B	100	20

# Market price fundamentals

## Market equilibrium



3 key ideas:

- *Equilibrium price*
- Price = *marginal cost*
- Market allocation *maximizes the total surplus*

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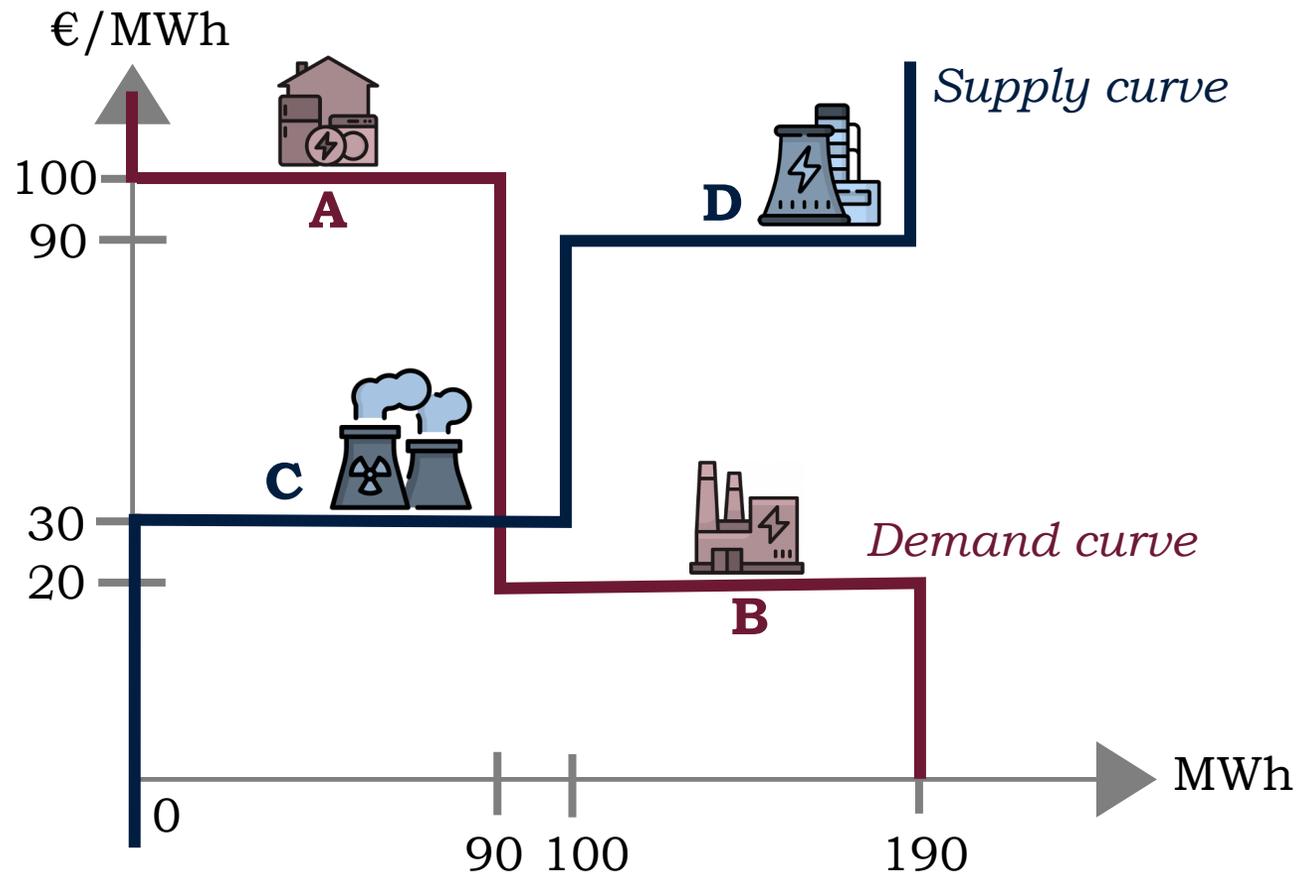
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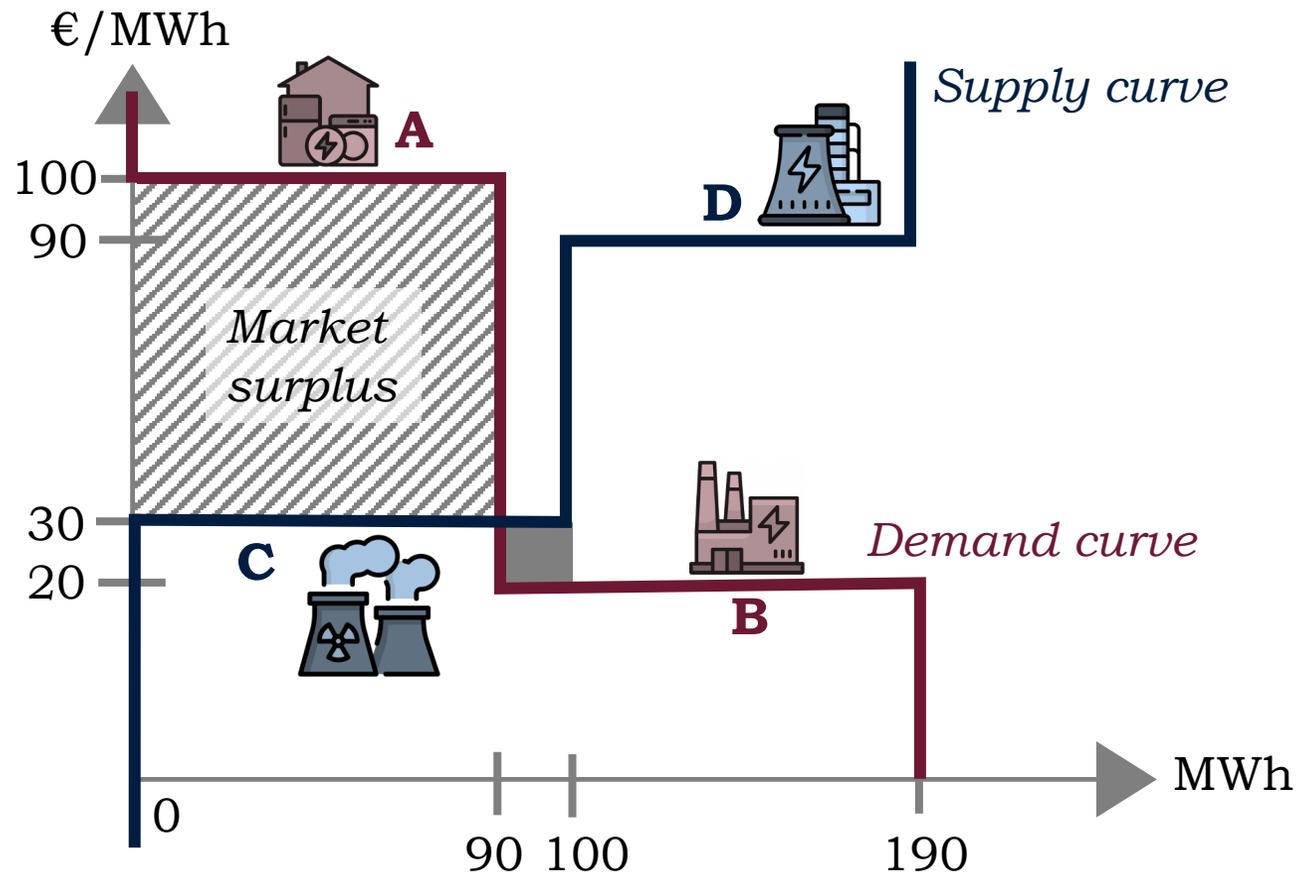
*Thesis contributions & conclusions*

# Pricing with indivisibilities



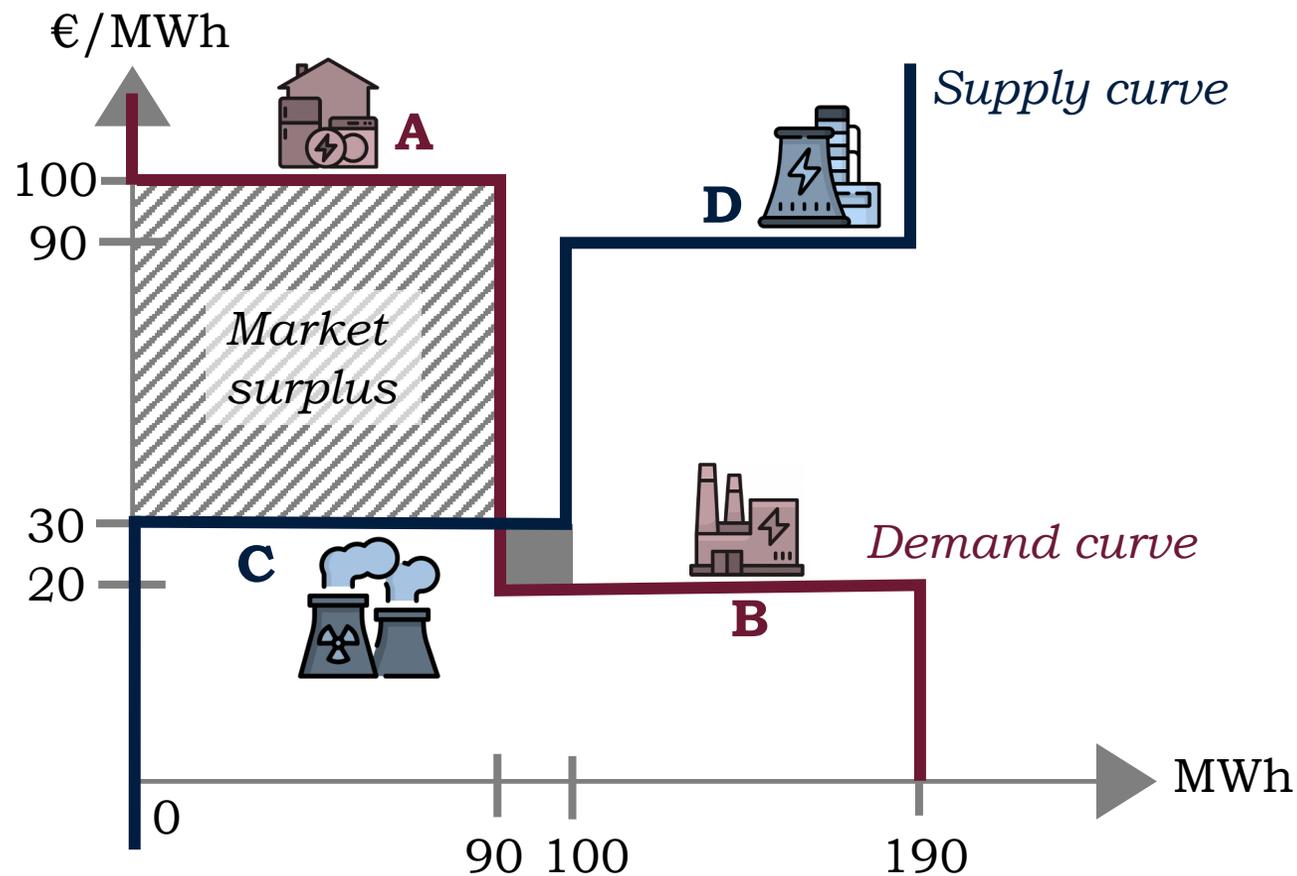
- What if **C** is *indivisible* (all-or-nothing)?

# Pricing with indivisibilities



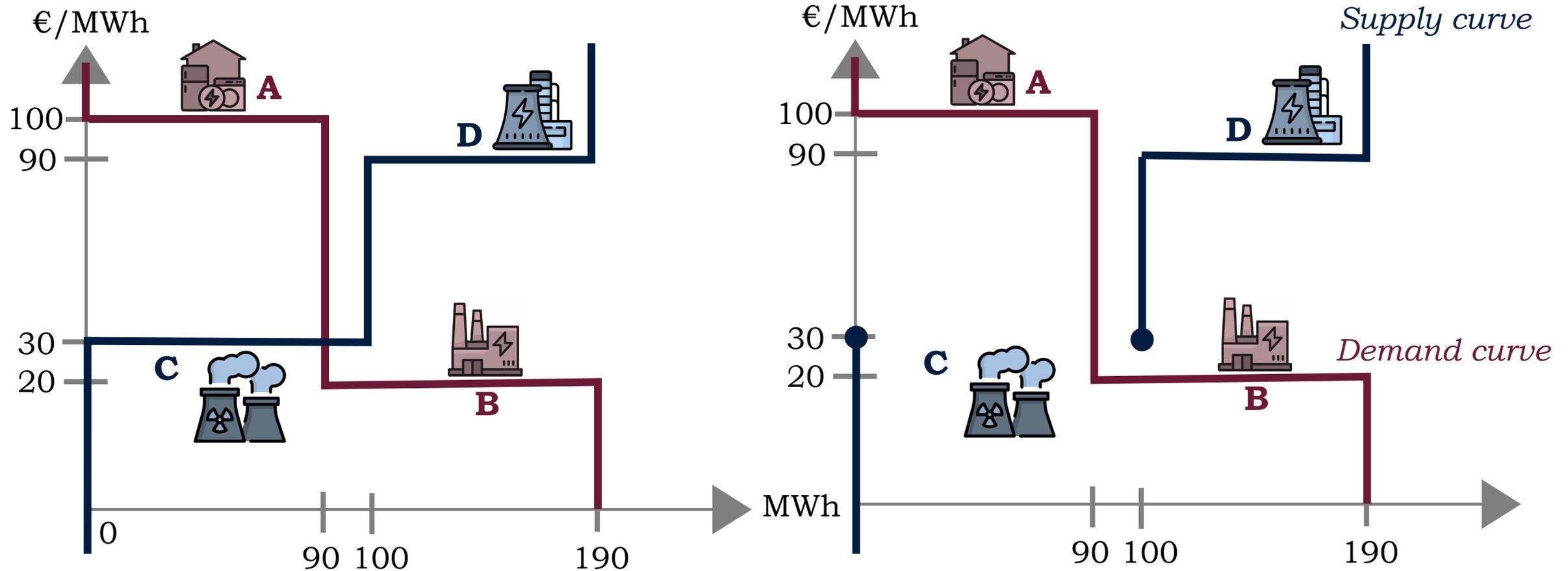
- What if **C** is *indivisible* (all-or-nothing)?
- Surplus maximizing solution: clear **A**, **C** and a fraction of **B**

# Pricing with indivisibilities



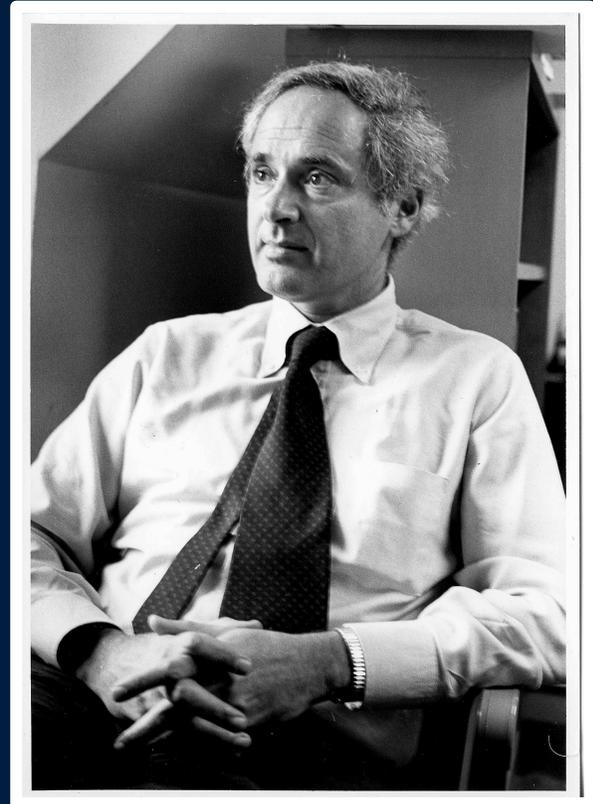
- What if **C** is *indivisible* (all-or-nothing)?
- Surplus maximizing solution: clear **A**, **C** and a fraction of **B**
- What is the price?
  - At 20€/MWh **C** is not willing to produce
  - At 30€/MWh, **B** is not willing to consume

# Pricing with indivisibilities



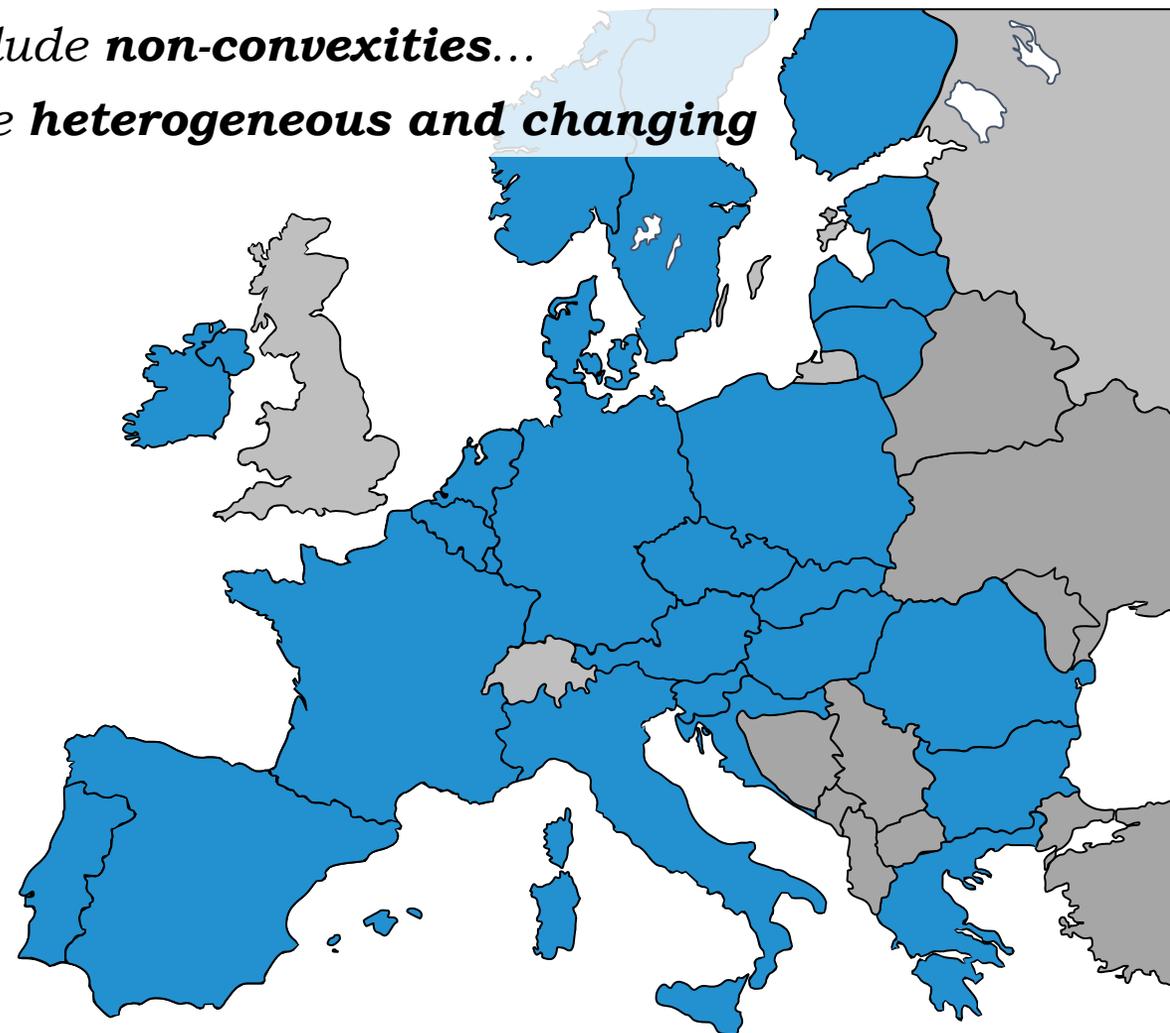
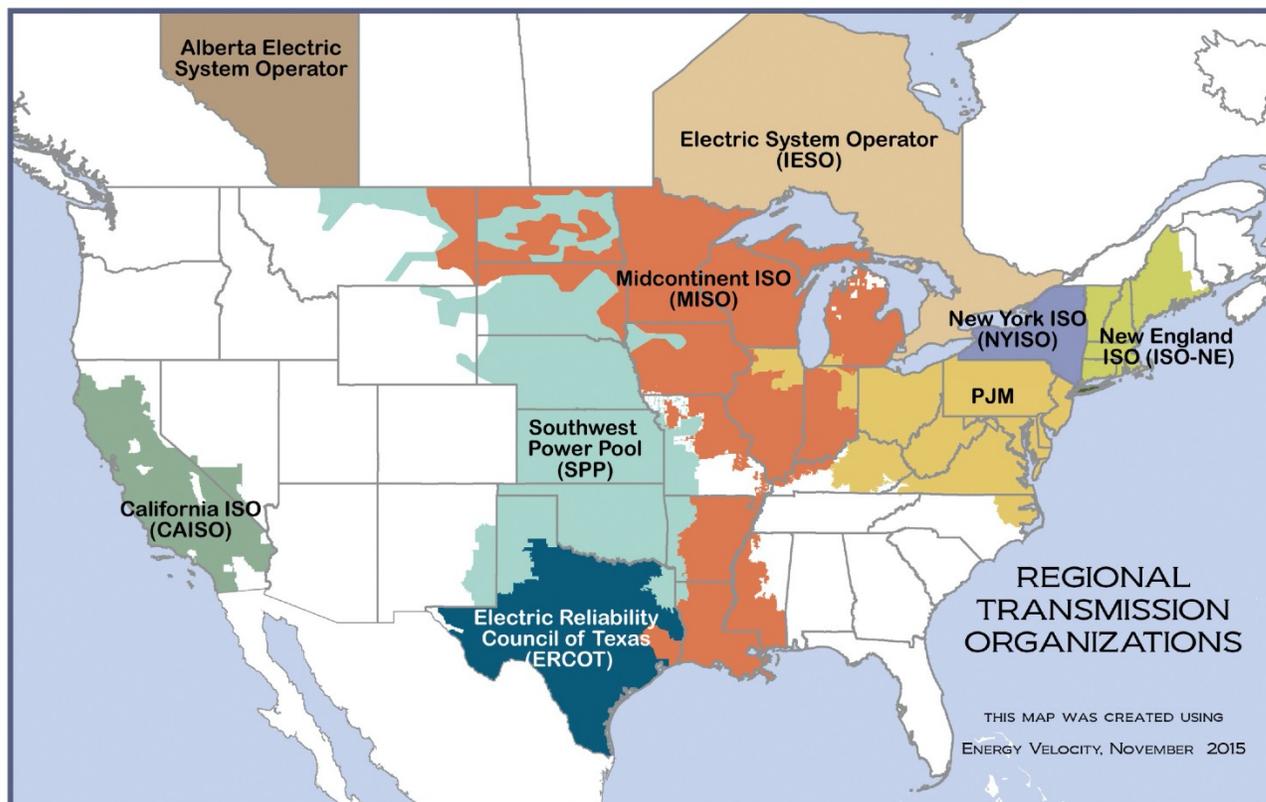
*“in the presence of indivisibilities in production,  
prices simply don’t do the jobs that they were  
meant to do”*

(Scarf, 1994)



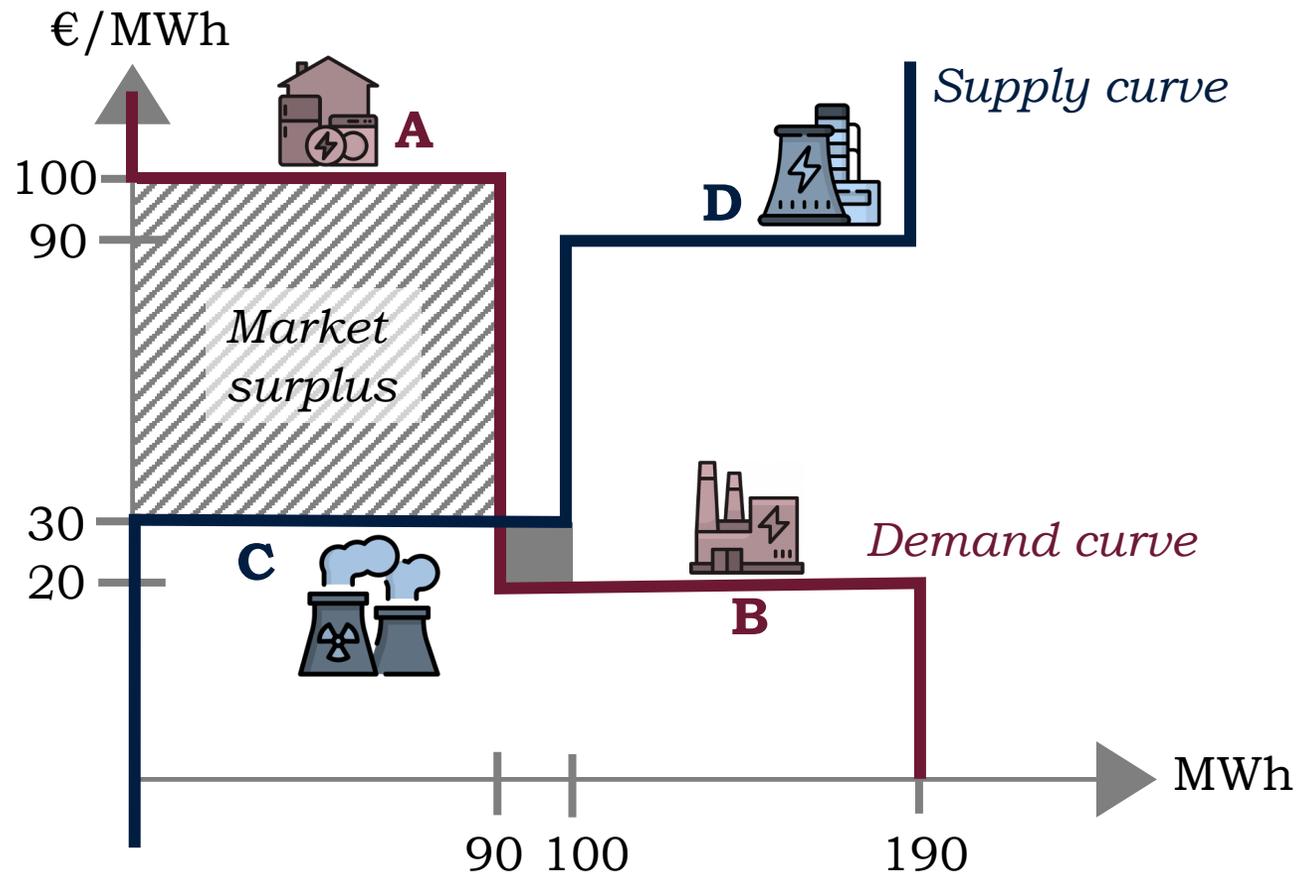
# This is a problem encountered in most electricity markets

All the US markets as well as the European market include **non-convexities...**  
...& the pricing policies that have been adopted are **heterogeneous and changing**



# Pricing with indivisibilities

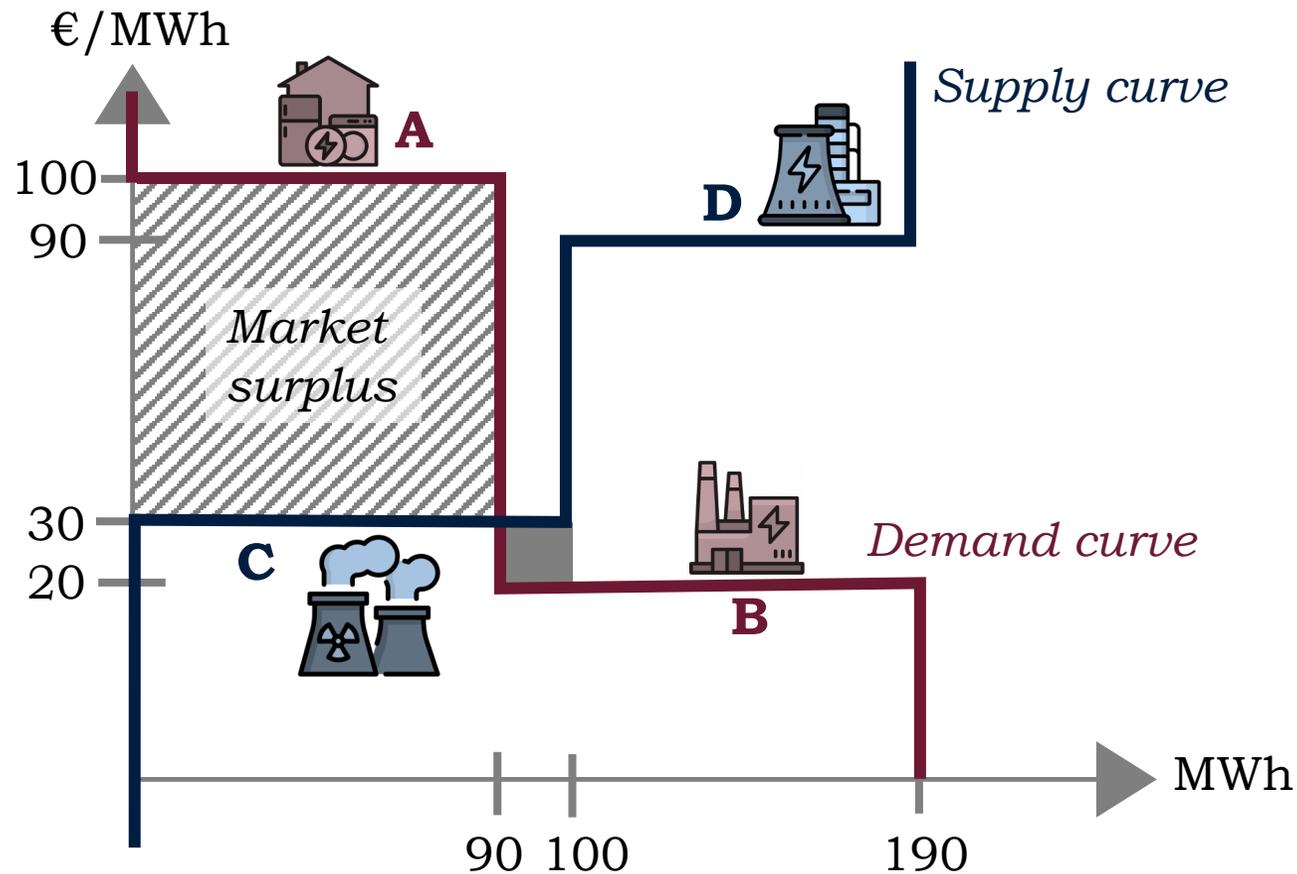
## Option 1: Marginal pricing



- Marginal price: 20€/MWh
- Supplier C: losing 10€/MWh
- Discriminatory payment of 1000€ to C

# Pricing with indivisibilities

## Option 2: “Convex Hull Pricing”



- Convex hull price: 30€/MWh
- Demand B: losing 10€/MWh
- Discriminatory payment of 100€ to B

→ Convex hull price minimize the incentives to deviates from the cleared allocation

→ Most of the thesis is based on this idea

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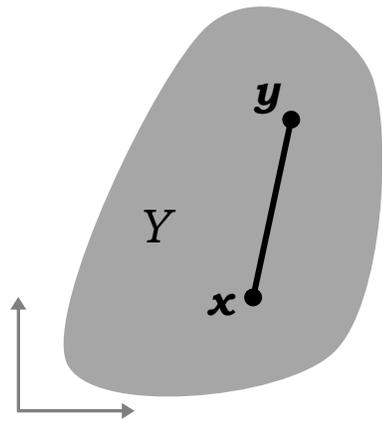
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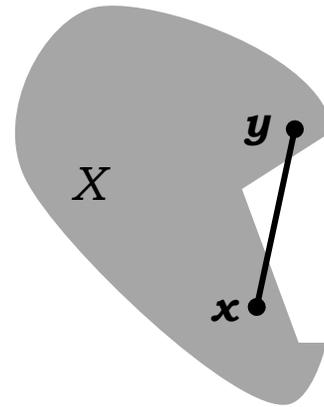
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*Thesis contributions & conclusions*

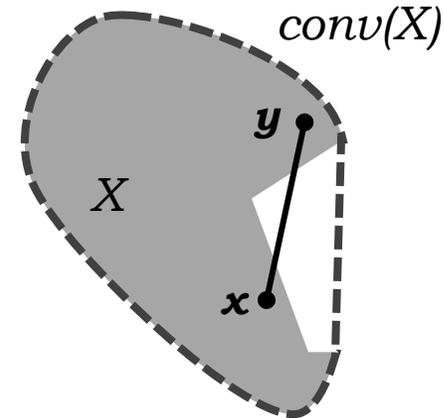
# 1 – Computing the *Convex Hull* Prices



**Convex set**

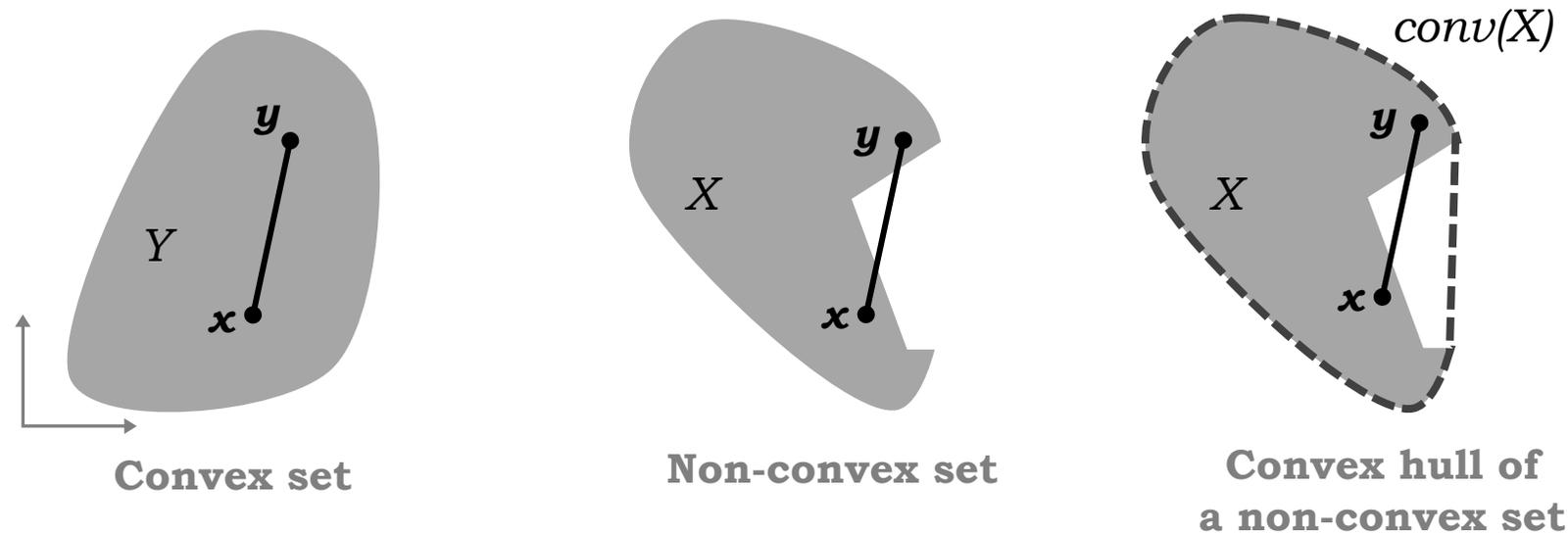


**Non-convex set**



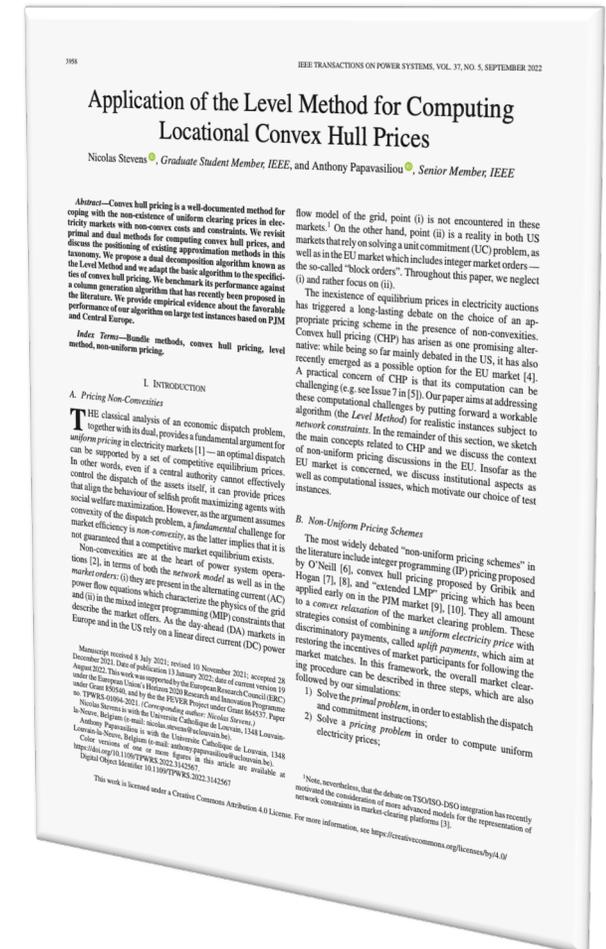
**Convex hull of  
a non-convex set**

# 1 – Computing the Convex Hull Prices



## Objectives & conclusions of the first thesis's contribution:

- Develop an **algorithm to compute the Convex Hull Prices**
- Our algorithm computes it on **realistic-size auction datasets** (1000 power units; 96 periods and up to 59 bidding zones)
- The largest instance is solved **in less than 10 minutes** (on a personal computer).
- **5 times faster** than the state-of-the art in the literature



# 2 – Economic properties of Convex Hull Pricing

## Objective & conclusions of the second thesis's contribution:

- **Theoretical formalization & cross-comparison** of several pricing approaches
- **Numerical simulations** on realistic auction datasets
- On some advantages of convex hull pricing:
  - Incentives of market participants
  - Distributional analysis across market participants (suppliers, consumers & SO)
  - Market size : the magnitude of the problem that, relative to the total surplus, shrinks with the market size
  - Consistency between cost minimization and “LOC” minimization
  - And more



# Conclusions & takeaways

- Electricity is a **key sector of our economies** and a sector experiencing **significant changes** with the energy transition
- Since **liberalization of the power sector** twenty+ years ago, the electricity price is the outcome of a **market**
- In presence of non-convexities, **pricing rule is not obvious**: “classic” *marginal pricing* fails to support the efficient allocation of resources
- Pricing the non-convexities in electricity auctions is a **long-lasting debate**
  - In the **US**: heterogeneous and changing policies
  - Recent discussions in **Europe** to reform the market pricing rule
- My thesis focused on the so-called “**convex hull pricing**” **approach**. Among others:
  - **Computational challenge**: develop an algorithm to compute it
  - **Economical properties**: several advantages of convex hull pricing

Thank you!

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